Homework Assignment #4: Due Monday, October 9

Please read Apostol, Sections 3.10, 3.11, 4.2, 4.6. Then please do the following exercises:

Part I. Apostol Chapter 3 (pp. 70–73): Exercise 2. Note: here, $d(n)$ denotes the number of positive divisors of n, and C denotes Euler's constant. Hint: first show that $d(n)/n = r * r(n)$, where $r(n) = 1/n$.

Part II.

(A) Prove that, if $s > 1$, then

$$\sum_{n=1}^{\infty} \frac{\mu(n)}{n^s} = \frac{1}{\zeta(s)}.$$

Hint: prove that

$$\zeta(s) \sum_{n=1}^{\infty} \frac{\mu(n)}{n^s} = 1,$$

by expanding the zeta function as a sum over m.

(B) Apostol Chapter 3 (pp. 70–73): Exercises 4(a), 5(a). (You may want to use part (A) directly above.)

(C) (This is easy.) Use parts (A,B) above to verify the first "=" given at the bottom of p. 70.

Part III. Apostol Chapter 4 (pp. 101–105): Exercise 20.

Part IV.

(1) Prove that, if

$$\sum_{n \le x} a(n) = Ax^u \log^v x + O(x^u \log^{v-1} x)$$

for $a(n)$ an arithmetic function, A a constant, and $u, v > 1$, then

$$\sum_{n \le x} \frac{a(n)}{n} = A\left(1 + \frac{1}{u-1}\right) x^{u-1} \log^v x + O(x^{u-1} \log^{v-1} x).$$

(2) Deduce the appropriate conclusion regarding $\sum a(n)/n$ if, in the hypotheses of problem **(1)** above, we replace "$u > 1$" with "$u = 1$."