

Homework Assignment #1: Due Friday, September 11

Please read Chapter 1 of Apostol. Then please do the following exercises:

Part I. Apostol Chapter 1 (pp. 21–23): Exercises 2, 11, 16, 19, 20.

Part II. (a) Use mathematical induction to prove that, for any positive integer n , the product of any n integers of the form $4\ell + 1$ (where ℓ is a positive integer) is itself of the form $4\ell + 1$.

(b) Show that there are infinitely many primes of the form $4\ell + 3$ (for ℓ a positive integer). You may want to use the result of part (a) of this exercise.

Hints:

- **Exercise 11:** Consider $n = 5k + r$ for $r = 0, 1, 2, 3, 4$. For the case $k = 0$, note that $(5k)^4 + 4$ factors. For the other cases, write things in terms of $r^4 + 4$, and compute the latter explicitly.
- **Exercise 16:** Try the contrapositive.
- **Exercise 19:** Induction.

Note: In completing this and any homework assignment, please adhere to the Guidelines for written assignments under the ADDITIONAL IMPORTANT COURSE INFORMATION banner on our course page – write in complete sentences, etc.