Please express all probabilities as decimals to four decimal places.

Suppose an event happens, on average, λ times on an interval of given length. Here we relate the probability that it happens k times on such an interval to the probability that it happens twice as many times on an interval that's twice as long.

Recall: suppose an event happens, on average, λ times per interval of a given extent, and X is the number of times that the event *actually* happens on a random interval of such an extent. Then we say "X is $P(\lambda)$," and we have the formula

$$P(X = k) = \frac{\lambda^k}{k!} e^{-\lambda} \qquad (k = 0, 1, 2, 3, \ldots). \qquad (*)$$

1. According to

`https://earthquakelist.org/japan/tokyo/tokyo/#major-latest-earthquakes`,

on average, 0.7 earthquakes of magnitude 4 or greater are registered within 300 km of Tokyo every day. So, if X is the number of such earthquakes per day, then X is $P($___0.7___$)$ (fill in the blank).

2. Using your answer to the previous problem, find the probability that there are exactly 2 earthquakes of magnitude 4 or greater registered within 300 km of Tokyo on a randomly chosen day.

$$P(X = 2) = \frac{0.7^2}{2!} e^{-.7} = 0.1217.$$

3. Now let Y be the number of earthquakes of magnitude 4 or greater registered within 300 km of Tokyo *in a two day period.*

Fill in the blanks. Since, on average, 0.7 earthquakes of magnitude 4 or greater are registered within 300 km of Tokyo every day, it follows that, on average, ___1.4___ earthquakes of magnitude 4 or greater are registered within 300 km of Tokyo every two-day period. In other words, Y is $P($___1.4___$)$.

4. Using your answer to the previous problem, find the probability that there are exactly 4 earthquakes of magnitude 4 or greater registered within 300 km of Tokyo on a randomly chosen two-day period.

$$P(Y = 4) = \frac{1.4^4}{4!} e^{-1.4} = 0.0395.$$

5. True or false: 4 earthquakes in two days is just as likely as 2 earthquakes in one day. Explain how you know.

False, because $P(Y=4) \neq P(X=2)$.

6. For X as in problem 1 above, compute all of the following probabilities (you already found one of these in one of the problems above):

$$P(X=0) = \frac{.7^0}{0!} e^{-.7} = 0.4966.$$

$$P(X=1) = \frac{.7^1}{1!} e^{-.7} = 0.3476.$$

$$P(X=2) = \frac{.7^2}{2!} e^{-.7} = 0.1217.$$

$$P(X=3) = \frac{.7^3}{3!} e^{-.7} = 0.0284.$$

$$P(X=4) = \frac{.7^4}{4!} e^{-.7} = 0.0050.$$

7. Fill in the blanks: There are various ways in which we can have 4 earthquakes over a 2 day period. *Either* there are no earthquakes on the first day *and* 4 on the second, *or* there is 1 earthquake on the first day *and* __3__ on the second, *or* there are __2__ earthquakes on the first day *and* __2__ on the second, *or* there are 3 earthquakes on the first day *and* __1__ on the second, *or* there are __4__ earthquakes on the first day *and* __0__ on the second. (If an earthquake straddles two days, assign it to one day or the other in some way.)

In other words, in terms of the random variables X and Y described above (and assuming earthquakes are __independent__ and __mutually__ exclusive),

$$\begin{aligned} P(Y=4) = P(X=0)\cdot P(X=4) &+ P(\underline{\quad X \quad}=1)\cdot P(X=3) \\ &+ P(X=\underline{\quad 2 \quad})\cdot P(X=\underline{\quad 2 \quad}) + P(\underline{\quad X \quad}=\underline{\quad 3 \quad})\cdot P(X=1) \\ &+ P(X=\underline{\quad 0 \quad})\cdot P(\underline{\quad X \quad}=4). \end{aligned}$$

8. Use your answers to the previous two problems to compute $P(Y = 4)$ (again). Your answer should match the answer to one of the above problems, at least up to some roundoff error.

$$\begin{aligned} P(Y = 4) &= 0.4966 \cdot 0.0050 + 0.3476 \cdot 0.0284 + 0.1217 \cdot 0.1217 + 0.0284 \cdot 0.3476 + 0.0050 \cdot 0.4966 \\ &= 0.0395. \end{aligned}$$

9. Answer in words, or in formulas, or however you think best: Suppose X is the number of times an event occurs every 10 seconds, and Y is the number of times it occurs every 20 seconds. Express $P(Y = 5)$ in terms of the appropriate values of $P(X = k)$.