EXAM 1: COMPLETELY RANDOM PRACTICE PROBLEMS

1. Consider events A, B, and C, in a sample space S, such that

$$P(A) = 0.3, \qquad P(B) = 0.4, \qquad P(C) = 0.2, \qquad P(A|C) = 0.5.$$

(a) Find $P(A \cup B)$, assuming A and B are *mutually exclusive.*

$$P(A \cup B) = P(A) + P(B) = 0.3 + 0.4 = 0.7.$$

(b) Find $P(A \cup B)$, assuming A and B are *independent.*

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = P(A) + P(B) - P(A)P(B)$$
$$= 0.3 + 0.4 - 0.3 \cdot 0.4 = 0.7 - 0.12 = 0.58.$$

(c) Find $P(AC)$.

$$P(AC) = P(C)P(A|C) = 0.2 \cdot 0.5 = 0.1.$$

(d) Find $P(A \cup C)$. Do not assume that A and C are mutually exclusive or independent.

$$P(A \cup C) = P(A) + P(C) - P(AC) = 0.3 + 0.2 - 0.1 = 0.4.$$

2. In the Lotto 7/30 lottery, seven distinct numbers are picked at random from the numbers 1,2, . . . , 30. What is the probability that the number 23 will be picked?

There are $\binom{30}{7}$ ways of choosing 7 numbers out of the 30. The number of these ways that will have a number 23 among the seven numbers is $\binom{29}{6}$. So the probability of there being a 23 among the seven numbers picked is

$$\frac{\binom{29}{6}}{\binom{30}{7}} = \frac{7}{30} = 0.2333\ldots \approx 23.33\%.$$

Note that, in the above, we are not keeping track of order. We could also do the problem by keeping track. Here's how. The number of possible ways of picking seven numbers is now

$$30 \cdot 29 \cdot 28 \cdot 27 \cdot 26 \cdot 25 \cdot 24.$$

How many of these ways contain a 23? Well first of all, there are seven different places to put the 23 among the seven numbers chosen. Once this is done, there are

$$29 \cdot 28 \cdot 27 \cdot 26 \cdot 25 \cdot 24$$

ways of choosing the other six numbers. So the probability of there being a 23 among the seven numbers picked is

$$\frac{7 \cdot 29 \cdot 28 \cdot 27 \cdot 26 \cdot 25 \cdot 24}{30 \cdot 29 \cdot 28 \cdot 27 \cdot 26 \cdot 25 \cdot 24} = \frac{7}{30} = 0.2333\ldots \approx 23.33\%.$$

3. A jar contains a combination of red, blue, and orange marbles, where some marbles are inscribed with the letter "W". Of the marbles in the jar, 2/5 are red marbles, 1/5 are blue marbles, and 2/5 are orange marbles. Also, 10% of the red marbles have a "W" on them. Similarly, 20% of the blue marbles have a "W", and 50% of the orange marbles have a "W".

You pick a marble at random from the jar. What is the probability that the marble has a "W" inscribed on it?

$P(\text{marble has a W})$
$= P(\text{you draw a red marble}) \cdot P(\text{marble has a W given that you draw a red marble})$
$+ P(\text{you draw a blue marble}) \cdot P(\text{marble has a W given that you draw a blue marble})$
$+ P(\text{you draw an orange marble}) \cdot P(\text{marble has a W given that you draw an orange marble})$
$= \frac{2}{5} \cdot 0.1 + \frac{1}{5} \cdot 0.2 + \frac{2}{5} \cdot 0.5 = 0.28 = 28\%.$

4. The word sleeplessness has 13 letters total; 5 of these letters are s, 2 of these letters are l, 4 of these letters are e, one of these letters is p, and one of these letters is n.

Two letters are chosen at random from the word sleeplessness. Find the probability that:

(a) Both letters are an s.

$$P(\text{both are s}) = \frac{\binom{5}{2}}{\binom{13}{2}} = \frac{5}{39} = 0.1282.$$

(b) Both letters are an l.

$$P(\text{both are l}) = \frac{\binom{2}{2}}{\binom{13}{2}} = \frac{1}{78} = 0.0128.$$

(c) Both letters are an e.

$$P(\text{both are e}) = \frac{\binom{4}{2}}{\binom{13}{2}} = \frac{1}{13} = 0.0769.$$

(d) Both letters are the same.

$$P(\text{both are the same}) = 0.1282 + 0.0128 + 0.0769 = 0.2179.$$

5. How many different 13-letter strings can be made from the letters in sleeplessness?

$$\binom{13}{5, 2, 4, 1, 1} = \frac{13!}{5!\,2!\,4!\,1!\,1!} = 1{,}081{,}080.$$

6. A certain free throw shooter, when shooting three free throws in a row, makes their first shot 80% of the time, and their second 90% of the time. Assume that their first two shots are independent of each other.

(a) (8 points) Find the probability that they hit their first two shots.

$$P(\text{both shots are made}) = P(\text{first is made}) \cdot P(\text{second is made})$$
$$= 0.8 \cdot 0.9 = 0.72 = 72\%.$$

(b) (6 points) Suppose this shooter hits all three free throws 60% of the time. Find the probability that they hit their third shot, given that they hit their first two. Do NOT assume that the third free throw is independent of the first two.

$$P(\text{third shot is made given that first two are made}) = \frac{P(\text{all three are made})}{P(\text{first two are made})}$$
$$= \frac{0.6}{0.72} = 0.833\ldots \approx 83.33\%.$$