EXAM 1: MORE COMPLETELY RANDOM PRACTICE PROBLEMS

1. Consider events A, E, F, and G, in a sample space S, such that

$$P(A) = 0.8, \qquad P(E) = 0.2, \qquad P(E \cup F) = 0.5, \qquad P(G) = 0.3, \qquad P(F|G) = 0.6.$$

(a) Find $P(A \cup G)$, assuming A and G are *independent.*

$$\begin{aligned} P(A \cup G) &= P(A) + P(G) - P(AG) = P(A) + P(G) - P(A)P(G) \\ &= 0.8 + 0.3 - 0.8 \cdot 0.3 = 0.86. \end{aligned}$$

(b) Find $P(F)$, assuming E and F are *mutually exclusive.*

$$P(F) = P(E \cup F) + P(E) = 0.5 - 0.2 = 0.3.$$

(c) Find $P(FG)$.

$$P(FG) = P(G)\,P(F|G) = 0.3 \cdot 0.6 = 0.18.$$

(d) Find $P(AFG)$, assuming A is independent from FG.

$$P(AFG) = P(A)\,P(FG) = 0.8 \cdot 0.18 = 0.144.$$

2. A database of trivia questions has easy, medium, and hard questions. Half (50%) of the questions are easy, 30% are medium, and 20% are hard. You have a 70% chance of answering an easy question correctly, a 50% chance of answering a medium one correctly, and 20% chance of answering a hard one correctly.

You pick a question at random from the database. You win the game if you answer that question correctly, and lose otherwise. What is the probability of winning?

$$\begin{aligned} P(\text{win}) &= P(\text{easy question})|P(\text{win given easy question}) \\ &\quad + P(\text{medium question})|P(\text{win given medium question}) \\ &\quad + P(\text{hard question})|P(\text{win given hard question}) \\ &= 0.5 \cdot 0.7 + 0.3 \cdot 0.5 + 0.2 \cdot 0.2 = 0.54 = 54\%. \end{aligned}$$

3. A jar contains a combination of red, blue, and orange marbles, where some marbles are inscribed with the letter "W". Of the marbles in the jar, 2/5 are red marbles, 1/5 are blue marbles, and 2/5 are orange marbles. Also, 10% of the red marbles have a "W" on them. Similarly, 20% of the blue marbles have a "W", and 50% of the orange marbles have a "W".

You pick a marble at random from the jar. What is the probability that the marble has a "W" inscribed on it?

$$\begin{aligned} P(\text{W}) &= P(\text{red marble})|P(\text{W given red marble}) \\ &+ P(\text{blue marble})|P(\text{W given blue marble}) \\ &+ P(\text{orange marble})|P(\text{W given orange marble}) \\ &= \frac{2}{5} \cdot 0.1 + \frac{1}{5} \cdot 0.2 + \frac{2}{5} \cdot 0.5 = 0.28 = 28\%. \end{aligned}$$

4. 18 socks total lie mixed up in a drawer: 8 blue socks, 4 red socks, 1 yellow sock, and 5 green socks. You grab two socks at random without looking. Find the probability that:

(a) Both socks are blue.

$$P(\text{both blue}) = \frac{\binom{8}{2}}{\binom{18}{2}} = 0.183007.$$

(b) Both socks are red.

$$P(\text{both red}) = \frac{\binom{4}{2}}{\binom{18}{2}} = 0.0392157.$$

(c) Both socks are green.

$$P(\text{both green}) = \frac{\binom{5}{2}}{\binom{18}{2}} = 0.0653595.$$

(d) Both socks have the same color.

$$P(\text{both same color}) = 0.183007 + 0.0392157 + 0.0653595 = 0.287582.$$

5. What's the probability that, of the 27 people in this class (including Dr. Slam), at least two share a birthday? (Assume that there are no February 29 birthdays, and that all other birthdays are equally likely.)

First, what's the probability that no two share a birthday? Well, first choose one person, who has a certain birthday. The probability that the second person chosen has a different birthday is 364/365. The probability that the third person chosen has a different birthday from the first two is 363/365. The probability that the fourth person chosen has a different birthday from the first three is 362/365. And so on down to the 27th person; the probability that they have a different birthday from the first 26 is $(365-26)/365 = 339/365$.

So the probability of no two sharing a birthday is

$$\frac{364}{365}\cdot\frac{363}{365}\cdot\frac{362}{365}\cdots\frac{339}{365}.$$

So the probability of at least two sharing a birthday is

$$1-\frac{364}{365}\cdot\frac{363}{365}\cdot\frac{362}{365}\cdots\frac{339}{365} = 0.626859 \approx 62.69\%.$$

6. You watch 10 pairs of socks, each pair a different color from the other pairs, and after washing, 6 socks are lost. Which is more likely: (a) the best case scenario (where the 6 lost were all in pairs, so 7 pairs remain intact), or (b) the worst case scenario (where all 6 lost socks are of different colors, so only 4 intact pairs remain)?

(a) There are $\binom{20}{6}$ ways of choosing 6 socks from the total of 20 socks. How many of these ways result in the best case scenario? Well, there are $\binom{10}{3}$ different ways of choosing which 3 pair get lost. So

$$P(\text{worst case scenario}) = \frac{\binom{10}{3}}{\binom{20}{6}} = 00309598 \approx 0.31\%.$$

(b) How many of the $\binom{20}{6}$ ways of choosing 6 socks from the total of 20 socks result in the worst case scenario? Well, first we choose which 6 colors get lost: there are $\binom{10}{6}$ ways to do this. Then, from each color lost, we choose one of the two socks: there are 2 ways to do this for each of the 6 colors. So

$$P(\text{worst case scenario}) = \frac{\binom{10}{6}\cdot 2^6}{\binom{20}{6}} = 0.346749 \approx 34.67\%.$$

Sadly, the worst case scenario is more than 100 times as likely as the best case scenario.