

EXAM 1: COMPLETELY RANDOM PRACTICE PROBLEMS

1. Consider events A , B , and C , in a sample space S , such that

$$P(A) = 0.3, \quad P(B) = 0.4, \quad P(C) = 0.2, \quad P(A|C) = 0.5.$$

- (a) Find $P(A \cup B)$, assuming A and B are *mutually exclusive*.
 - (b) Find $P(A \cup B)$, assuming A and B are *independent*.
 - (c) Find $P(AC)$.
 - (d) Find $P(A \cup C)$. Do not assume that A and C are mutually exclusive or independent.
2. In the Lotto 7/30 lottery, seven distinct numbers are picked at random from the numbers $1, 2, \dots, 30$. What is the probability that the number 23 will be picked?
3. A jar contains a combination of red, blue, and orange marbles, where some marbles are inscribed with the letter “W”. Of the marbles in the jar, $2/5$ are red marbles, $1/5$ are blue marbles, and $2/5$ are orange marbles. Also, 10% of the red marbles have a “W” on them. Similarly, 20% of the blue marbles have a “W”, and 50% of the orange marbles have a “W”.

You pick a marble at random from the jar. What is the probability that the marble has a “W” inscribed on it?

4. The word sleeplessness has 13 letters total; 5 of these letters are s, 2 of these letters are l, 4 of these letters are e, one of these letters is p, and one of these letters is n.

Two letters are chosen at random from the word sleeplessness. Find the probability that:

- (a) Both letters are an s.
 - (b) Both letters are an l.
 - (c) Both letters are an e.
 - (d) Both letters are the same.
5. How many different 13-letter strings can be made from the letters in sleeplessness?
6. A certain free throw shooter, when shooting three free throws in a row, makes their first shot 80% of the time, and their second 90% of the time. Assume that their first two shots are independent of each other.
- (a) Find the probability that they hit their first two shots.
 - (b) Suppose this shooter hits all three free throws 60% of the time. Find the probability that they hit their third shot, given that they hit their first two. Do NOT assume that the third free throw is independent of the first two.