## Applications of SDM, concluded.

### A) Population proportion.

Recall: the proportion $p$ of a population that has a given property can be estimated by the 95% confidence interval

$$\left( \hat{p} - 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \right), \qquad (*)$$

where $\hat{p}$ is the sample proportion and $n$ is the sample size.

The quantity $1.96\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$

is called the margin of error, or standard error.

**Example 1.**

The Berkman Center at Harvard interviewed 588 teens, and found that 159 of them had 500 or more Facebook friends.

Find a:

(a) 95%,

(b) 98%

confidence interval for the proportion $p$ of teens with $\geq 500$ Facebook friends.

Solution.

Our "point estimate" $\hat{p}$ is

$$\hat{p} = \frac{159}{588} = 0.2704$$

We compute that

$$\sqrt{\frac{0.2704(1-0.2704)}{588}} = 0.0262.$$

So: (a) the 95% interval, by (*), is

$$(0.2704 - 1.96 \cdot 0.0262,\ 0.2704 + 1.96 \cdot 0.0262)$$
$$= (0.2191,\ 0.3217)$$

(b) By (*) with 2.33 in place of 1.96, the 98% interval is

$$(0.2704 - 2.33 \cdot 0.262,\ 0.2704 + 2.33 \cdot 0.0262)$$
$$= (0.2094,\ 0.3314).$$

Note: the quantity

$$1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

appearing in (*) is called the <u>margin of error</u> (at the 95% level).

FACT (you can prove this with calculus): $\hat{p}(1-\hat{p})$ is <u>at most</u> $= 1/4$. SO: at the 95%

level, the margin of error is at most

$$1.96\sqrt{\frac{1/4}{n}} = \frac{1.96}{\sqrt{4n}} = \frac{1.96}{2\sqrt{n}} = \frac{0.98}{\sqrt{n}}.$$

Example 2.

In estimating the proportion of the population that's disgusted with politics, we want a margin of error, at the 95% level, that's no more than 3 percentage points. How large a sample size $n$ do we need?

Solution.

We want

$$\frac{0.98}{\sqrt{n}} \le 0.03,$$

$$0.98 \le 0.03\sqrt{n},$$

$$\sqrt{n} \geq \frac{0.98}{0.03},$$

$$n \geq \left(\frac{0.98}{0.03}\right)^2 = 1067.11$$

So $n \geq 1068$ will suffice.