

**Please express all probabilities as decimals to four decimal places.**

Suppose an event happens, on average,  $\lambda$  times on an interval of given length. Here we relate the probability that it happens  $k$  times on such an interval to the probability that it happens twice as many times on an interval that's twice as long.

Recall: suppose an event happens, on average,  $\lambda$  times per interval of a given extent, and  $X$  is the number of times that the event *actually* happens on a random interval of such an extent. Then we say “ $X$  is  $P(\lambda)$ ,” and we have the formula

$$P(X = k) = \frac{\lambda^k}{k!} e^{-\lambda} \quad (k = 0, 1, 2, 3, \dots). \quad (*)$$

1. According to

<https://earthquakelist.org/japan/tokyo/tokyo/#major-latest-earthquakes>,

on average, 0.7 earthquakes of magnitude 4 or greater are registered within 300 km of Tokyo every day. So, if  $X$  is the number of such earthquakes per day, then  $X$  is  $P(\underline{0.7})$  (fill in the blank).

2. Using your answer to the previous problem, find the probability that there are exactly 2 earthquakes of magnitude 4 or greater registered within 300 km of Tokyo on a randomly chosen day.

$$P(X = 2) = \frac{0.7^2}{2!} e^{-0.7} = 0.1217.$$

3. Now let  $Y$  be the number of earthquakes of magnitude 4 or greater registered within 300 km of Tokyo *in a two day period*.

Fill in the blanks. Since, on average, 0.7 earthquakes of magnitude 4 or greater are registered within 300 km of Tokyo every day, it follows that, on average, 1.4 earthquakes of magnitude 4 or greater are registered within 300 km of Tokyo every two-day period. In other words,  $Y$  is  $P(\underline{1.4})$ .

4. Using your answer to the previous problem, find the probability that there are exactly 4 earthquakes of magnitude 4 or greater registered within 300 km of Tokyo on a randomly chosen two-day period.

$$P(Y = 4) = \frac{1.4^4}{4!} e^{-1.4} = 0.0395.$$

5. True or false: 4 earthquakes in two days is just as likely as 2 earthquakes in one day. Explain how you know.

False, because  $P(Y = 4) \neq P(X = 2)$ .

6. For  $X$  as in problem 1 above, compute all of the following probabilities (you already found one of these in one of the problems above):

$$P(X = 0) = \frac{.7^0}{0!} e^{-.7} = 0.4966.$$

$$P(X = 1) = \frac{.7^1}{1!} e^{-.7} = 0.3476.$$

$$P(X = 2) = \frac{.7^2}{2!} e^{-.7} = 0.1217.$$

$$P(X = 3) = \frac{.7^3}{3!} e^{-.7} = 0.0284.$$

$$P(X = 4) = \frac{.7^4}{4!} e^{-.7} = 0.0050.$$

7. Fill in the blanks: There are various ways in which we can have 4 earthquakes over a 2 day period. *Either* there are no earthquakes on the first day *and* 4 on the second, *or* there is 1 earthquake on the first day *and* 3 on the second, *or* there are 2 earthquakes on the first day *and* 2 on the second, *or* there are 3 earthquakes on the first day *and* 1 on the second, *or* there are 4 earthquakes on the first day *and* 0 on the second. (If an earthquake straddles two days, assign it to one day or the other in some way.)

In other words, in terms of the random variables  $X$  and  $Y$  described above (and assuming earthquakes are independent and mutually exclusive),

$$\begin{aligned} P(Y = 4) &= P(X = 0) \cdot P(X = 4) + P(\text{X} = 1) \cdot P(X = 3) \\ &\quad + P(X = \text{2}) \cdot P(X = \text{2}) + P(\text{X} = \text{3}) \cdot P(X = 1) \\ &\quad + P(\text{X} = 4) \cdot P(X = \text{0}). \end{aligned}$$

8. Use your answers to the previous two problems to compute  $P(Y = 4)$  (again). Your answer should match the answer to one of the above problems, at least up to some roundoff error.

$$\begin{aligned}P(Y = 4) &= 0.4966 \cdot 0.0050 + 0.3476 \cdot 0.0284 + 0.1217 \cdot 0.1217 + 0.0284 \cdot 0.3476 + 0.0050 \cdot 0.4966 \\&= 0.0395.\end{aligned}$$

9. Answer in words, or in formulas, or however you think best: Suppose  $X$  is the number of times an event occurs every 10 seconds, and  $Y$  is the number of times it occurs every 20 seconds. Express  $P(Y = 5)$  in terms of the appropriate values of  $P(X = k)$ .

$$\begin{aligned}P(Y = 5) &= P(X = 0) \cdot P(X = 5) + P(X = 1) \cdot P(X = 4) \\&\quad + P(X = 2) \cdot P(X = 3) + P(X = 3) \cdot P(X = 2) \\&\quad + P(X = 4) \cdot P(X = 1) + P(X = 5) \cdot P(X = 0) \\&= 2(P(X = 0) \cdot P(X = 5) + P(X = 1) \cdot P(X = 4) \\&\quad + P(X = 2) \cdot P(X = 3)) \\&= \sum_{k=0}^5 P(X = k) \cdot P(X = 5 - k) \\&= 2 \sum_{k=0}^2 P(X = k) \cdot P(X = 5 - k).\end{aligned}$$