Chapter 4: Random variables.

Definition 1.

Given a sample space S, a random variable (rv) X is a function on S.

That is, an rv assigns numbers to outcomes of an experiment.

Example 1.

Roll two 6-sided dice; write, as usual,

$$S = \{11, 12, 13, \ldots, 66\}.$$

One rv we could define is

X = sum on dice.

E.g. $X(13) = 4$, $X(52) = 7$, $X(16) = 7$, etc.

X takes values 2 through 12.

We could define a different rv

Y = product on dice.

E.g. $Y(13) = 3$, $Y(52) = 10$, etc.

Y takes values

1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 16, 18, 20, 24, 25, 30, 36.

Example 2

Take 3 free throws; let Z = number of hits. Write H for hit and M for miss. Then

$$Z(HHM) = 2, \quad Z(MMM) = 0, \text{ etc.}$$

Defintion 2.

if X is an rv and x a possible value of X, write $P(X=x)$ for the probability that X attains the value x.

Example 3.

In Example 1 above, assuming fair dice,

$$P(X=8) = \frac{|\{26,35,44,53,62\}|}{36} = \frac{5}{36} = 0.13\overline{8},$$

etc.

Similarly,

$$P(Y=12) = \frac{|\{26,34,43,62\}|}{36} = \frac{4}{36} = 0.\overline{1},$$

etc.

Example 4.

In Example 2 above, assume we have an 80% free throw shooter, and that all free throws are independent.

Then

$P(Z=3) = P(\{HHH\}) = .8 \cdot .8 \cdot .8 = 0.512.$

$P(Z=0) = P(\{MMM\}) = .2 \cdot .2 \cdot .2 = 0.008.$

Challenge question: $P(Z=1) = ?$
$P(Z=2) = ?$

Solution: $P(Z=1) = P(\{HTT, THT, TTH\})$
$= .8 \cdot .2 \cdot .2 + .2 \cdot .8 \cdot .2 + .2 \cdot .2 \cdot .8$
$= 0.096.$

Similarly, $P(Z=2) = 0.384.$

Super-challenge question: If n independent free throws are taken, each with probability p of going in, and Z is the number of hits, what is

$$P(Z=r), \quad \text{for } 0 \le r \le n?$$

(Answer: $P(Z=r) = \binom{n}{r} p^r (1-p)^{n-r}$)

Definition 3.

Given an rv X, the probability mass function (pmf) for X is just the set of all possible values of $P(X = x)$.

E.g. in Example 4 above, we computed the pmf for Z.

Example 5.

For the rv. Z of Example 4, we can describe the pmf in (at least) three different ways:

(a) By listing the values:

$$\left.\begin{aligned} P(Z=0) &= 0.008 \\ P(Z=1) &= 0.096 \\ P(Z=2) &= 0.384 \\ P(Z=3) &= 0.512 \end{aligned}\right\}$$

Note that the probabilities add up to 1.

(b) By a formula:

$$P(Z = z) = \binom{3}{z} 0.8^{z} 0.2^{3-z} \quad (0 \le z \le 3).$$

(see super-challenge question above).

(c) In a histogram:

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Free throws made out of 3, 80% shooter