Conditional probability, continued.

The multiplication rule

$$P(A_1 A_2) = P(A_1)P(A_2 | A_1)$$

generalizes as follows:

$$\boxed{\begin{aligned} &P(A_1 A_2 A_3 \dots A_n) \\ &= P(A_1)P(A_2|A_1)P(A_3|A_1A_2)P(A_4|A_1A_2A_3)\cdots \\ &\cdot P(A_n | A_1 A_2 A_3 \cdots A_n). \end{aligned}}$$

Generalized Multiplication Rule

Example 1.

4 ping-pong players and 4 tennis players are selected for a pickleball tournament. These 8 players are randomly paired for the first round.

Find P(no tennis player pairs with a tennis player).

Solution

Number the tennis players 1, 2, 3, 4. Define

A_i: player i is paired with a ping-pong player. We're looking for

$$P(A_1 A_2 A_3 A_4).$$

Now

$$P(A_1) = 4/7, \quad P(A_2|A_1) = 3/5$$

$$P(A_3|A_1A_2) = 2/3, \quad P(A_4|A_1A_2A_3) = 1$$

So

$$P(A_1A_2A_3A_4) = \frac{4}{7}\cdot\frac{3}{5}\cdot\frac{2}{3}\cdot 1 = \frac{8}{35} = 0.22857.$$

In a population of 10,000 people, 1% are infected with COVID-19. All 10,000 people are tested, using a test that has a 2% false positive rate (2% of those who are uninfected will test positive), and a 7% false negative rate. Complete the table, and the conclusion below.

Number of people	Infected	Uninfected	Total
Test positive	93 (true positive)	198 (false positive)	291
Test negative	7 (false negative)	9702 (true negative)	9709
Total	100	9900	10000

CONCLUSION: Even though the test has a false positive rate of only 2%, out of all people who test positive for COVID-19, only

$$\frac{93}{291} \approx 32\ \%$$ are actually infected!