Activity: Coins, counting, and probability

Imagine an experiment where you flip six fair coins, and observe whether each coin lands heads or tails. Imagine that the outcome of this experiment is recorded as a string of six letters, where each letter is an H (for heads) or a T (for tails). (The first letter indicates how the first coin lands, and so on. For example, HTHTTH.)

1. How many possible outcomes are there? Please explain.

There are two ways for each of the six coins to land, so there are $2^6 = 64$ total possible outcomes.

2. Write down, explicity (as strings of T's and H's), all outcomes in which exactly two coins land heads.

HHTTTT, HTHTTT, HTTHTT, HTTTHT, HTTTTH,

THHTTT, THTHTT, THTTHT, THTTTH, TTHHTT,

TTHTHT, TTHTTH, TTTHHT, TTTHTH, TTTTHH

3. What's the probability that, on a given trial of this experiment, exactly two coins land heads? You don't need any formal probability formulas or ideas here. Just use common sense, and your answers to Exercises 1 and 2 above. Express your answer as a decimal or a percent.

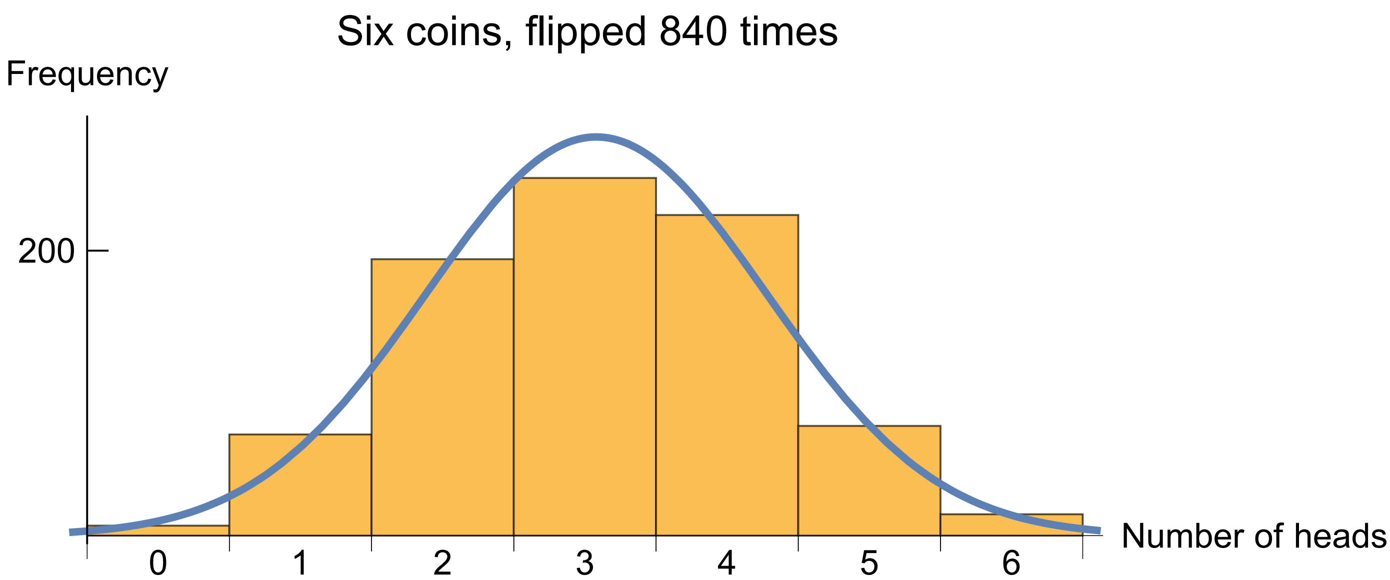
There are 15 outcomes with exactly two heads, and 64 outcomes altogether, so the probability of an outcome with exactly two heads is

$$\frac{15}{64} = 0.234375 \approx 23.44\%.$$

4. Let's actually do the experiment. Flip your six coins, and put a tally mark in the corresponding slot on the "frequency table" below. (That is: if you get three heads, put a tally mark in the "3" row of the "Frequency" column, and so on.) Repeat this experiment *at least one hundred* times, tallying your results each time in the table below. **In the table, please mark down not only your tally marks, but also the total tally (count), in each row.**

Number of heads	Frequency
0	7
1	71
2	194
3	251
4	225
5	77
6	15

5. On the axes below, draw a histogram depicting the information from your above frequency table. PLEASE BE NEAT. (Don't forget to fill in your number of trials, where it says "Six coins, flipped ____ times".)



6. Does the number of times that exactly two coins landed heads, in your histogram, agree (roughly) with your answer to Exercise 3 above? Please explain.

In our experiment, we had exactly two coins land heads 194 out of 840 times. This means that exactly two coins landed heads $194/840 = 0.230952 \approx 23.10\%$ of the time, which is really quite close to our theoretical probability of 23.44%.

7. Suppose the experiment were to flip 30 coins, instead of 6, at a time. What's the probability that exactly 17 of them land heads?

HINT: Don't actually try to write down all possible outcomes with 17 H's!! Instead: reason as follows: first, how many possible outcomes are there in total? (Hint: it's similar to Exercise 1 above.) Then: how many of these outcomes have exactly 17 H's? (Hint: it involves a binomial coefficient.)

If you can't get your calculator to give you an exact answer, at least write down what the answer is in terms of factorials and powers of 2.

There are $\binom{30}{17}$ ways in which exactly 17 of the coins can land heads, and 2^{30} possible length-30 strings of H's and T's total. So the probability of exactly 17 coins of 30 landing heads is

$$\frac{\binom{30}{17}}{2^{30}} = \frac{\frac{30!}{17!\,13!}}{1,073,741,824} = 0.111535 \approx 11.15\%.$$

8. Suppose you were to flip 30 coins, record the number of heads, repeat this experiment 100,000 times, and draw a histogram like the one in Exercise 5 above. What do you think the approximate *shape* of this histogram be? Please explain.

We'd expect a bell curve (a normal cure, to be precise), centered at 15. Here's a simulation: