Imagine an experiment where you roll two fair dice, and observe the number appearing on each.

We're going to study the following question from three different perspectives:

Given that one of the dice lands on 6, what's the probability that the other does as well (meaning both land on 6)?

1. (First approach.) Fill in the blank (express the answer as a fraction and as a percent): we reason as follows. What one die does has nothing to do with what the other one does. So the probability of one die landing on 6 should be the same regardless of what the other one does. So

$$P(\text{one die lands on 6 given that the other does})$$

$$= P(\text{one die lands on 6}) = \underline{\quad 1/6 = 16.67\% \quad},$$

since a given die has 6 possible outcomes.

2. (Second approach.) We think about outcomes and events. As usual, let's denote an outcome of rolling two dice by a two-digit string, like 23 or 61.

(a) Write down all outcomes where at least one of the dice lands on a 6. I've started for you, to show you what I mean.

Answer: $16, 26,$ $\underline{36, 46, 56, 66, 65, 64, 63, 62, 61}$.

How many such outcomes are there? Answer: $\underline{11}$

(b) Out of all the above outcomes where at least one die lands on 6, how many have both dice landing on 6 ?

Answer: $\underline{1}$

(c) If we divide the number of outcomes where *both* land on 6 (see part (b) above) by the number of outcomes where at least one does (see part (a) above), we should get a good idea of the *probability* of both landing on 6, *given* that at least one does. So go ahead and divide. What do you get, as a fraction and as a percent?

Answer: $\underline{1/11=9.09\%}$

3. (Third approach.) Presumably, your answers to Exercises 1 and 2 above disagree. So let's do an experiment to try to understand which one is correct.

(a) Roll your two dice.

(b) If (at least) one of the dice lands on 6, then put a tally mark in the first row of the frequency table below, where it says "(At least) one die lands on 6." Otherwise, do nothing.

(c) If you *did* put a tally mark in the first row – that is, if at least one of your dice landed on 6 – AND if the other die landed on 6 as well (in other words, if both dice landed on 6), then put a tally mark in the second row of the frequency table below, where it says "Both dice land on 6." Otherwise, do nothing.

Now repeat (a,b,c) until you have *at least* 75 tally marks in the first row (the "(At least) one die lands on 6" row). (If getting 75 tally marks there happens quickly, keep going until you have 100.)

Answer:

Event	Frequency
(At least) one die lands on 6.	79
Both dice land on 6.	656

(d) Now, divide the number of tally marks in the *second* row (the "Both dice land on 6" row) by the number of tally marks in the first row (the "(At least) one die lands on 6" row). What is the number you get? (Express as a fraction and as a percent.)

Answer: $79/656 = 12.04\%$.

(e) Explain why the answer you got to part (d) above should give you some sort of idea of the probability

P(both dice land on 6 given that at least one does).

Explanation: The above number tells us the *proportion* of rolls with two 6's, out of all rolls with at least one. That *proportion* should, at least approximately, reflect the *probability* of getting two 6's given that you get at least one.

IMPORTANT NOTE: every group in class except for one got an answer to part (d) above that was somewhere around 8% to 11%. The fact that the average over all groups is closer to 12% is because there was a single group whose answer was closer to 22%! That number threw everything else off. In certain situations one might have considered that group's data to constitute an *outlier*, and that data would not be considered.

If one uses data from all groups *except* that one group, one gets a proportion of 10.25% instead of 12.04%.

4. Looking at the different numbers for

P(both dice land on 6 given that (at least) one does)

as determined in Exercises 1, 2, and 3 above, do you have any thoughts as to which answer(s) is/are best? Why?

Exercise 2 is best. The correct answer for

P(both dice land on 6 given that (at least) one does)

really is $1/11 = 0.91\%$. We calculated this by carefully considering sample spaces and events, instead of using intuition.

If we had, say, a green die and a blue die, and we asked for

P(the blue die lands on 6 given that the green one does) or

P(the green die lands on 6 given that the blue one does),

then the correct answer *would* be $1/6 = 16.67\%$. But that's not what we're asking.

Our experimental data was not as close to the actual answer as I would have expected, but again, if one removes the outlier data, then one gets an empirical probability of 10.25%, which is fairly close to our theoretical probability of 9.1%.