Hints for Selected Exercises, HW #7

Assignment.

Chapter 4 problems, starting on p. 175:

Problems 41, 43, 44, 46, 49, 50.

Note: We will use the shorthand "X is $B(n,p)$" to mean X is a binomial random variable with n trials and, in each trial, $P(\text{success}) = p$. (To express the same thing, the book says "X is a binomial random variable with parameters (n,p).")

Exercise 41: Here, X is $B(5,1/3)$. You want to find $P(X = 4) + P(X = 5)$, where X is the number correct.

Exercise 43: (a) By independence, the probability that both A and B get a given question correct is $.4 \cdot .7 = .28$. So, if Z is the number that both get correct, then Z is $B(10,0.28)$.

(b)

$$P(A \text{ or } B \text{ is correct}) = P(A \text{ is correct}) + P(B \text{ is correct}) - P(A \text{ and } B \text{ are correct}).$$

Exercise 44: Let X be the number of digits that are received incorrectly. Then X is $B(5,.2)$. The question is asking: what is $P(X > 2)$?

Exercise 46: We have

$$\begin{aligned}&P(\text{pass with 3 examiners})\\ &= P(\text{on day})P(\text{pass with 3 examiners given on day})\\ &+ P(\text{off day})P(\text{pass with 3 examiners given off day}).\end{aligned}$$

Now $P(\text{on day})$ and $P(\text{off day})$ can be computed from the given information. Moreover, let X be the number of examiners, out of 3, who pass the student on an on day. Then X is $B(3,.8)$, and $P(\text{pass with 3 examiners given on day})$ is $P(X \geq 2)$. Similarly for Y, the number of examiners who pass the student on an off day. So from all of this info, you should be able to compute $P(\text{pass with 3 examiners})$.

Next, do the similar thing with 5 examiners, and compare.

Exercise 49 This is kind of a binomial-within-a-binomial problem.

First, let X be the number defective in a *single* package. Then X is $B(10,.01)$. To find the probability that a single package is returned, you need to find $P(X > 1)$. (Hint: to find this, it might be easier to compute $1 - P(X = 0) - P(X = 1)$.)

Now denote the probability that you just computed above (the probability that a single package can be returned) by p. Let Y be the number that can be returned if 3 packages are bought: then Y is $B(3,p)$.

Exercise 50: **(a)** If X is the number of heads using Coin 1, then X is $B(10, .4)$. If Y is the number of heads using Coin 2, then Y is $B(10, .7)$.

We have

$$\begin{aligned} P(7 \text{ heads}) = &\ P(\text{Coin 1})P(7 \text{ heads given Coin 1}) \\ &+ P(\text{Coin 2})P(7 \text{ heads given Coin 2}). \end{aligned}$$

(b)

$$P(7 \text{ heads} \mid \text{first is heads}) = \frac{P(7 \text{ heads and first is heads})}{P(\text{first is heads})}.$$

Now

$$\begin{aligned} P(\text{first is heads}) &= P(\text{Coin 1})P(\text{first is heads given Coin 1}) \\ &\quad + P(\text{Coin 2})P(\text{first is heads given Coin 2}) \\ &= \frac{1}{2} \cdot .4 + \frac{1}{2} \cdot .7 = .55. \end{aligned}$$

Moreover,

$$\begin{aligned} &P(7 \text{ heads and first is heads}) \\ &= P(\text{Coin 1})P(7 \text{ heads and first is heads given Coin 1}) \\ &+ P(\text{Coin 2})P(7 \text{ heads and first is heads given Coin 2}). \end{aligned}$$

Now what is $P(7 \text{ heads and first is heads given Coin 1})$? Its

$$P(\text{first is heads for Coin 1}) \text{ times } P(6 \text{ of the remaining 9 are heads for Coin 1}).$$

The probability of the first being heads is .4. Also

$$P(6 \text{ of the remaining 9 are heads for Coin 1}) = \binom{9}{6}(.4)^6(.6)^3.$$

Similarly, you can compute $P(7 \text{ heads and first is heads given Coin 2})$.