Hints for Selected Exercises, HW #4

Assignment.

Chapter 3, pages 103–111: Problems 1, 2, 5, 9, 12, 14, 16, 23.

Exercise 2: Please note that they are making a distinction between the two dice, so the event "the first one lands on 6" includes the outcome 62, but not the outcome 26.

Exercise 5: Let E be the event that the second two are black and F the event that the first two are white. By equation (2.2), page 59,

$$P(EF) = P(F) \cdot P(E|F).$$

Now since all outcomes are equally likely (because the balls are chosen randomly), and because there are originally 6 white and 9 black balls, we have

$$P(F) = \frac{\text{number of ways of choosing 2 balls from 6}}{\text{number of ways of choosing 2 balls from 15}}.$$

Next, given that the first two balls are white, there are 13 balls remaining of which 9 are black, so

$$P(E|F) = \frac{\text{number of ways of choosing 2 balls from 9}}{\text{number of ways of choosing 2 balls from 13}}.$$

Use combinations (that is, $\binom{n}{r}$) to figure out the numbers you need.

Exercise 9: Let A be the event that the ball selected from urn A is white, B the event that the ball selected from urn B is white, and C the event that the ball selected from urn C is white. Let T be the event that exactly two of the selected balls are white. Then

$$P(A|T) = \frac{P(AT)}{P(T)}.$$

Now note that saying *exactly* two balls are white amounts to saying that either A and B happen but C doesn't, or A and C happen but B doesn't, or B and C happen but A doesn't. So

$$T = ABC^c \cup AB^cC \cup A^cBC.$$

Similarly,

$$AT = ABC^c \cup AB^cC.$$

You should be able to compute $P(A)$, $P(B)$, and $P(C)$ based on how many white balls, and total balls, are in each urn. Using the fact that ABC^c, AB^cC, and A^cBC are mutually exclusive (why?), and A, B, and C are independent, you should be able to get everything you need.

Exercise 12: List the cards in order of ascending value. For example, if the value of card C is smallest, followed by the value of card A, followed by the value of card B, then write CAB. By just counting outcomes, you should be able to compute $P(\text{value on } A < \text{value on } B)$ as well as

$$P(\text{value on } A < \text{value on } B \text{ and value on } A < \text{value on } C).$$

Then you can use equation (2.1) on page 58.

Exercise 14: We have

$$P(E_1E_2E_3E_4) = P(E_1)P(E_2|E_1)P(E_3|E_1E_2)P(E_4|E_1E_2E_3).$$

To compute $P(E_1)$, first note that there are $\binom{52}{13}$ choices for the first 13-card hand. How many of these choices have exactly one ace? Well, there are 4 ways of choosing which ace, and for each such choice, there are $\binom{48}{12}$ choices for the remaining 12 cards (since the remaining 12 all have to be non-aces).

To compute $P(E_2|E_1)$ note that, if E_1 has been achieved, then there are $\binom{39}{13}$ choices for the second 13-card hand, of which $3 \cdot \binom{36}{12}$ have exactly one ace (there are three choices left for which ace, and then the remaining 12 cards can be neither an ace nor one of the other 12 cards chosen for the first hand, so there are $52 - 4 - 12$ cards from which to choose).

So how many possibilities does this leave for the third hand, and of these possibilities, how many have exactly one ace? And then the remaining hand?

I believe the answer you get in the end (which doesn't seem to be in the back of the book) is 0.105498.

Exercise 16: Let S and E be the obvious events here. We're trying to find $P(S|E)$.

Note that, by equation (2.1),

$$P(S|E) = \frac{P(SE)}{P(E)}. \tag{1}$$

But by equation (2.2),

$$P(ES) = P(S)P(E|S). \qquad (2).$$

Since ES is the same as SE, we can plug (2) into (1) to get

$$P(S|E) = \frac{P(S)P(E|S)}{P(E)}. \tag{3}$$

We are given $P(S)$, so if we can figure out what $P(E|S)/P(E)$ is, we're done.

To figure this out note that, by equation (3.1) on page 64, also known as Bayes's formula,

$$P(E) = P(E|S)P(S) + P(E|S^c)(1 - P(S)).$$

Into this formula, plug in the given facts $P(S) = .32$ and $P(E|S^c) = P(E|S)/2$, and do some algebra (show your work!) to find a *number* for $P(E|S)/P(E)$. As noted above, this should do it.

Exercise 23: The sample space has size 6^3.

(a) There are 6 possibilities for the blue die, then 5 for the yellow, then 4 for the red.

(b) This is much like Exercise 12 above, except with B, Y, and R instead of A, B, and C.