Solutions to Selected Exercises, HW #1

Assignment.

Chapter 1, pages 15–17: 1, 3, 4, 8, 10, 11, 15, 16, 17, 18, 22, 23.

Exercise 1: **(a)** How many different 7-place license plate numbers are possible if the first two places are for letters and the other 5 are for numbers?

(b) Repeat part (a) under the assumption that no letter or number can be repeated in a single license plate.

SOLUTION: **(a)** There are 26 choices for the first place, for each of which there are 26 choices for the second, then 10 for the third, then 10 for the fourth, and same for the fifth, sixth, and seventh places, yielding

$$26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 67{,}600{,}000$$

possible different plates.

(b) In this case, there are 26 possibilities for the first place, then 25 for the second, then 10 for the third, then 9 for the fourth, 8 for the fifth, 7 for the sixth, and 6 for the seventh, yielding

$$26 \cdot 25 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 = 19{,}656{,}000$$

possible different plates.

Exercise 4: John, Jim, Jay, and Jack have formed a band consisting of 4 instruments. If each of the boys can play all 4 instruments, how many different arrangements are possible? What if John and Jim can play all four instruments, but Jay and Jack can each only play piano and drums?

SOLUTION: We can assign instruments to John, Jim, Jay, and Jack in order. If each can play an instrument, then there are 4 choices for John, leaving 3 for Jim, and so on, yielding $4! = 24$ possibilities. If Jay and Jack can each only play piano and drums, then we have to reserve those two instruments for them, so there are only 2 choices for John, leaving only one choice for Jim, and then two choices (piano or drums) for Jay, and then the last instrument (drums or piano) for Jack. So in this case, there are $2 \cdot 1 \cdot 2 \cdot 1 = 4$ possible arrangements. (If it's easier, imagine assigning Jay and instrument first, then Jack, then John, then Jim.)

Exercise 10: In how many ways can 8 people be seated in a row if

(a) there are no restrictions on the seating arrangement?

(b) persons A and B must sit next to each other?

(c) there are 4 men and 4 women and no two men or 2 women can sit next to each other?

(d) there are 5 men and they must sit next to each other?

(e) there are 4 married couples and each couple must sit together?

SOLUTION: **(a)** $8! = 40{,}320$. **(b)** The AB pair can occupy either the first and second seats in the row, or the second and third, or the third and fourth, $\ldots$, or the seventh and eighth. There are 7 choices here, but for each of these choices, we can either have A on the left of the pair or B on the left. Once A and B are seated, there are 6! ways of seating the other 6. So there are

$$7 \cdot 2 \cdot 6! = 10{,}080$$

possibilities. **(c)** The four men must either be in seats 1, 3, 5, and 7, or in seats 2, 4, 6, and 8. For each of these two possibilities, the 4 men can be seated in 4! different orders. Then the 4 women can be seated in the remaining 4 seats, again in 4! possible ways. So the total number of possibilities is

$$2 \cdot 4! \cdot 4! = 1{,}152.$$

(d) The five men sit either in seats 1 through 5, or 2 through 6, or 3 through 7, or four through 8. For each of these possibilities, there are 5! seatings of the 5 men. There are then 3! possible seatings of the 3 women, yielding

$$4 \cdot 5! \cdot 3! = 2{,}880$$

possibilities. **(e)** There are 4! choices for where to put each couple, but for each of these 4! choices, each couple can be seated in two possible ways (either one of the two people in each couple can be seated on the left, and the other on the right). This yields

$$4! \cdot 2^4 = 384$$

possibilities.

Exercise 15: Consider a group of 20 people. If everyone shakes hands with everyone else, how many handshakes take place?

SOLUTION: Each of the 20 people shakes hands with 19 others. So the answer would be $20 \cdot 19$, except that were's overcounting; we've counted each handshake twice. So the correct answer is $(20 \cdot 19)/2 = 190$.

Exercise 17: A dance class consists of 22 students, of which 10 are women and 12 are men. If 5 men and 5 women are to be chosen and paired off, how many results are possible?

SOLUTION: There are $\binom{10}{5}$ ways of choosing the 5 women. Once such a choice is made, there are 12 men with whom the first woman may be paired, leaving 11 men who can be paired with the second woman, and so on, down to 8 choices for the fifth woman. This yields

$$\binom{10}{5} \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 = 23{,}950{,}080$$

pairings.

Another way of counting the same thing is as follows: choose the 5 women and then the 5 men; there are $\binom{10}{5} \cdot \binom{12}{5}$ ways of doing this. There are then 5 men who can be paired with the first woman, leaving four to be paired with the second, then 3 with the third, and so on, yielding 5! possible pairings of the five men with the 5 women. So the total number of pairings of 5 women out of 10 with 5 men out of 12 is

$$\binom{10}{5} \cdot \binom{12}{5} \cdot 5! = 23{,}950{,}080$$

(again).

Exercise 18: A student has to sell 2 books from a collection of 6 math, 7 science, and 4 economics books. How many choices are possible if

(a) both books are to be on the same subject,

(b) both books are to be on different subjects?

SOLUTION: (a) The two books are either math books, or science books, or economics books, so there are

$$\binom{6}{2} + \binom{7}{2} + \binom{4}{2} = 42$$

possibilities. **(b)** For each of the 3 possible pairs of subjects, choose one book from each of the 2 subjects. This yields

$$6 \cdot 7 + 6 \cdot 4 + 7 \cdot 4 = 94$$

possibilities.

Exercise 22: A person has 8 friends, of whom 5 will be invited to a party.

(a) How many choices are there if 2 of the friends are feuding and will not attend together?

(b) How many choices if 2 of the friends will only attend together?

SOLUTION: (a) There are $\binom{6}{5}$ choices in which neither of the feuding friends attend; and $2 \cdot \binom{6}{4}$ choices in which one of these two friends attends, but not both. (First choose which of the 2 feuding friends will attend, then choose 4 guests from the set of 6 that excludes the other feuding friend.) The total number of possibilities is then

$$\binom{6}{5} + 2 \cdot \binom{6}{4} = 36.$$

(b) Either both attend — there are $\binom{6}{3}$ ways for this to happen — or both don't — there are $\binom{6}{5}$ ways. The total is

$$\binom{6}{3} + \binom{6}{5} = 26$$

ways.