Solutions to Selected Exercises, HW #12

Assignment: Statistics Notes, Section 1.2, subsection H.

H.1.

$$95\% : (0.568, 0.702)$$
$$98\% : (0.556, 0.714)$$
$$99\% : (0.547, 0.723)$$

H.2. Let $f(\widehat{p}) = \widehat{p}(1-\widehat{p})$. Then $f'(\widehat{p}) = 1-2\widehat{p}$. This has a critical point at $\widehat{p} = 1/2$: it's a local maximum because $f''(\widehat{p}) = -2 < 0$. Since there's only one local maximum, it must be the global maximum. The value of f at this maximum is $1/2(1-1/2) = 1/4$, so indeed, the maximum value of $\widehat{p}(1-\widehat{p})$ is $1/4$. And again, this maximum is attained at $\widehat{p} = 1/2$.

H.3. (a) We want $0.98/\sqrt{n} \leq 0.02$. Solving for n gives $n \geq ((0.98/0.02)^2 = 2401$. So $n = 2401$ will suffice.

(b) We want $2.33/(2\sqrt{n}) \leq 0.02$. Solving for n gives $n \geq (2.33/(2 \cdot 0.02))^2 = 3393.06$. So $n = 3394$ will suffice.

(c) We want $2.58/(2\sqrt{n}) \leq 0.02$. Solving for n gives $n \geq (2.58/(2 \cdot 0.02))^2 = 4160.25$. So $n = 4161$ will suffice.

H.4. (a) Three of the intervals are centered at 0.2, but the interval $(0.1420, 0.2436)$ is not, so we eliminate that interval. Of the remaining three, the wider the interval, the higher the confidence level, so we have the following:

$$95\% : (0.1608, 0.2392)$$
$$98\% : (0.1534, 0.2466)$$
$$99\% : (0.1484, 0.2516)$$

(b) A confidence interval is centered on $\widehat{p}$, so we must have $\widehat{p} = 0.2$.

(c) The margin of error in the 95% interval is $0.2392 - 0.2 = 0.0392$. We solve:

$$1.96\sqrt{\frac{0.2(1-0.2)}{n}} = 0.0392,$$
$$\frac{1.96 \cdot 0.4}{\sqrt{n}} = 0.0392,$$
$$n = \left(\frac{1.96 \cdot 0.4}{0.0392}\right)^2 = 400.$$