1. A certain basketball player takes 30 shots in a game. Suppose the player is a 60% shooter (meaning 60% of their shots go in). What is the expected number of times that this player will hit two shots in a row, in such a game? Hint: let X_1 equal 1 if the first two shots go in, and 0 otherwise. Let X_2 equal 1 if the second and third shots go in, and 0 otherwise. And so on. Then let $X = X_1 + X_2 + ...$, and compute $E(X)$.

2. You pay \$6 to play a game where a fair die is rolled. You lose if the die lands on an even number, you receive \$9 if the die lands on a 1 or a 3, and you receive \$12 if it lands on a 5.

   (a) Find the probability mass function for your payoff X (meaning how much you receive minus the \$6 put in to play).

   (b) What are your expected winnings (meaning how much you receive minus the \$6 put in to play) from this game?

3. You pay \$5 to play the following game. You toss an unfair coin, with P(heads)$= 1/3$. If the coin lands heads, you choose two marbles at random from a jar containing 3 red marbles and 2 blue marbles. If the coin lands tails, you choose two marbles from a jar containing 4 red marbles and 2 blue marbles. You are then awarded \$15 if you end up with two red marbles; otherwise, you receive \$0.

   (a) Find the probability mass function for your payoff X (meaning how much you receive minus the \$5 put in to play).

   (b) What are your expected winnings (meaning how much you receive minus the \$5 put in to play) from this game?

4. Here we consider two similar scenarios. For both of these, recall that a standard deck of cards has 4 aces, and 48 cards that aren't aces.

   (A) We draw three cards *without replacement* at random from a standard deck. That is: each card that is drawn is left out of the deck before drawing the next card. (This is the usual way cards are drawn or dealt.) Let X be the number of aces drawn.

   (B) We draw three cards *with replacement* at random from a standard deck. This means: We draw the first card, record what kind of card it is (ace or not), and *put it back in the deck*. We then draw the second card, record what kind of card it is (ace or not), and *put it back in the deck*. Finally, we draw the third card, and record what kind of card it is (ace or not). Let Y be the total number of aces drawn.

   Here are the questions.

   (a) One of the above scenarios represents a binomial experiment, and one does not. Which is which? Why?

   (b) Compute the probability mass function for X (from scenario (A) above).

   (c) Compute $E[X]$.

   (d) This time, compute the probability mass function for Y (from scenario (B) above).

(e) Compute $E[Y]$.

(f) How do $E[X]$ and $E[Y]$ compare?

(g) Which is larger, $P(X = 0)$ or $P(Y = 0)$? Why does this make sense? That is, could you have predicted this without any computation, and if so, how?

(h) Compute $\mathrm{Var}[X]$ and $\mathrm{Var}[Y]$. How do they compare? Why does this make sense? That is, could you have predicted this without any computation, and if so, how?