More on sets:

universe, complement, Venn diagrams.

1) In some contexts, we assume that all sets in question are subsets of some universe U. E.g. if we're discussing properties of integers, we might stipulate that $U = \mathbb{Z}$.

2) Given a universe U and a set $A \subseteq U$, we define the <u>complement</u> $\bar{A}$ of A (in U) by

$$\bar{A} = U - A.$$

Examples:

(a) Let $U = \{a, b, c, d, e\}$.
Then

$$\overline{\{a, b\}} = \{c, d, e\},$$
$$\overline{\{c, d, e\}} = \{a, b\}$$

(note that, for any set A and universe U, $\bar{\bar{A}} = A$).

(b) Let $U = \mathbb{Z}$.
Then

$$\overline{\{\text{odd numbers}\}} = \{\text{even numbers}\},$$
$$\overline{3\mathbb{Z}} = (1 + 3\mathbb{Z}) \cup (2 + 3\mathbb{Z}),$$
$$\overline{\mathbb{N}} = \{\ldots, -3, -2, -1, 0\} = \{0\} \cup \{-n : n \in \mathbb{N}\},$$
$$\overline{\varnothing} = \mathbb{Z},$$
$$\overline{\mathbb{Z}} = \varnothing,$$

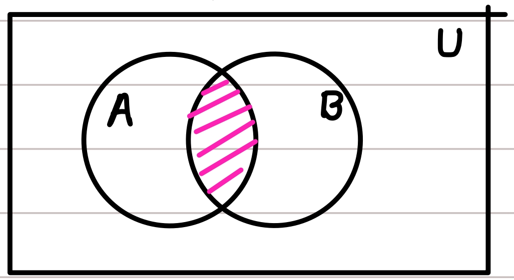
etc.

3) Venn diagrams.

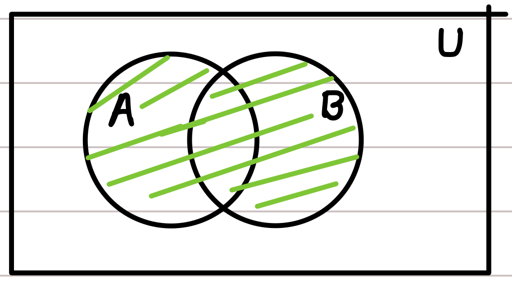
Depict the universe U as a box; sets are regions in the box.

Use Venn diagrams (with shading, if it helps) to:

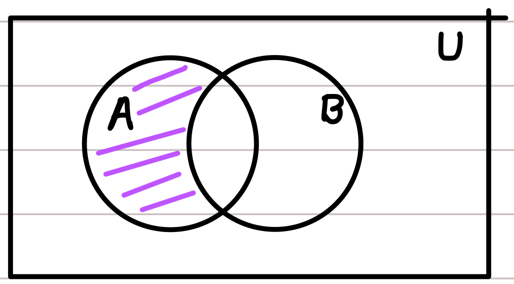
(a) Depict set operations. Examples:



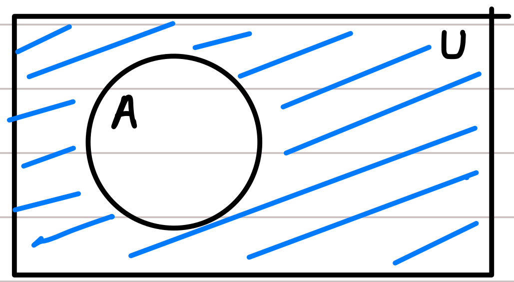
(i) $A \cap B$



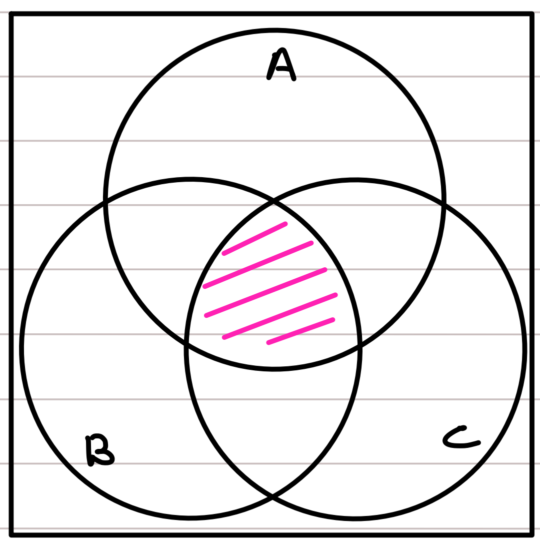
(ii) $A \cup B$



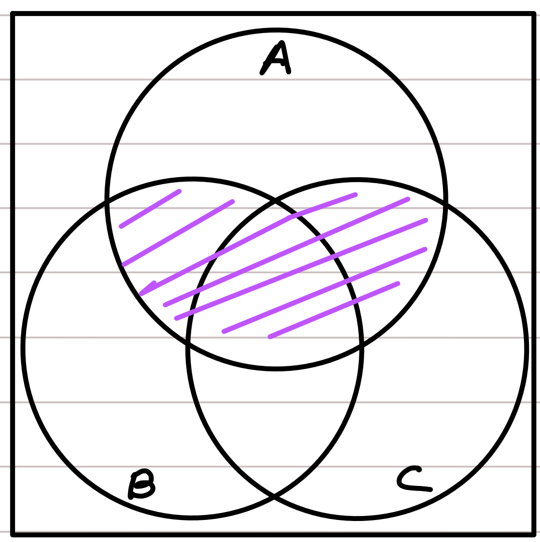
(iii) $A - B$



(iv) $\bar{A}$

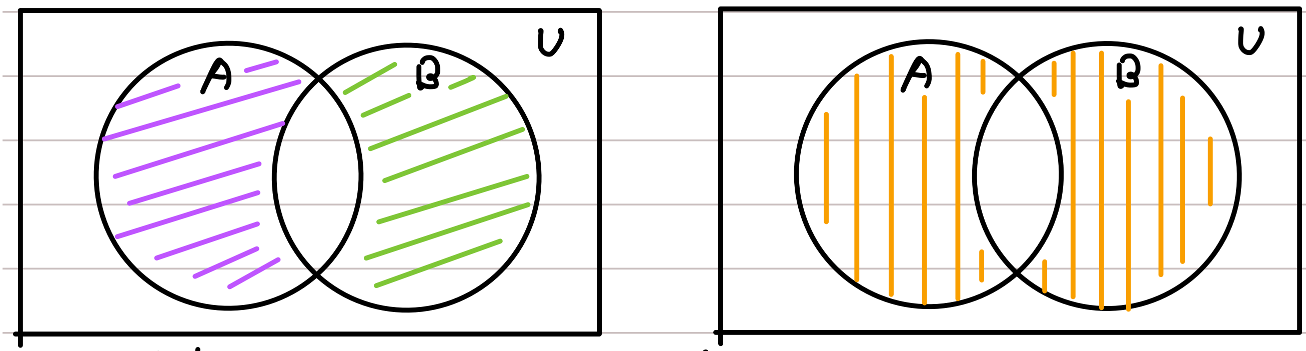


(v) $A \cap B \cap C$



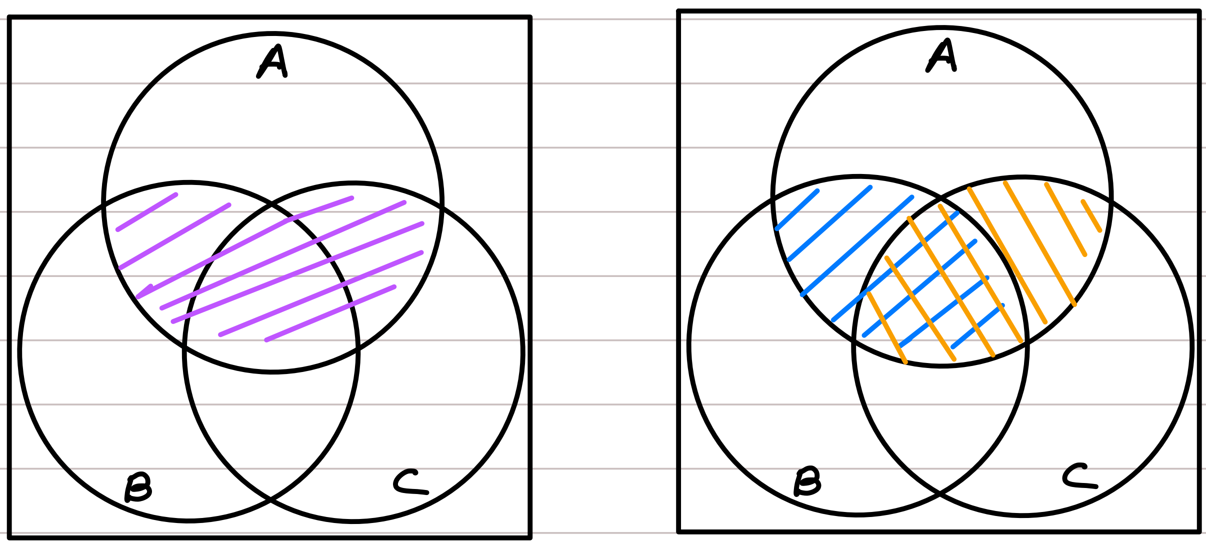
(vi) $A \cap (B \cup C)$

(b) Illustrate set <u>relations</u> (facts). Examples:



(i) Illustrates that

$$(A-B)\cup(B-A)=(A\cup B)-(A\cap B)$$



(ii) Illustrates that

$$A\cap(B\cup C) = (A\cap B)\cup(A\cap C)^{*}$$

(Illustrations are not proofs!)

*Note the analogy with the distributive law

$$a\cdot(b+c)=ab+ac \quad !$$

DIY: use Venn diagrams to illustrate $A\cup(B\cap C)$ and $(A\cup B)\cap(A\cup C)$. Are they the same?