Probability basics

Mathematically, probability is about sets (of things that can happen).

Some definitions, with examples.

1) Experiment: a phenomenon that can happen in various ways. For example:

Experiment I: flip three coins.
Experiment II: roll two dice.

2) Outcome: any of those ways.
E.g. for experiment I, we can describe outcomes as length-3 strings of T's and H's. For example, HHT or THH.

Experiment II: we can write an outcome as a two-digit number ij, where $1 \le i, j \le 6$. For example, 34 or 43.

3) Sample space: the set of all possible outcomes.

Experiment I: the sample space V is

$$V = \{HHH, HTH, HHT, HTT, THH, TTH, THT, TTT\}.$$

Experiment II: the sample space W is

$$W = \{\text{strings } ij : 1 \le i, j \le 6\}.$$

4) Event: a set of outcomes. So: an event is a <u>subset</u> of S.

Example: for Experiment I above, define events:

$F = \{$outcomes with at most one heads$\}$
$= \{HTT, THT, TTH, TTT\}$,
$G = \{$outcomes where the middle coin is tails$\}$
$= \{HTH, TTH, HTT, TTT\}$,
$H = \{$outcomes with at least two consecutive H's$\} = \{HHT, THH, HHH\}$.

For experiment II, define:

$A = \{$outcomes that sum to 7$\}$
$= \{16, 25, 34, 43, 52, 61\}$,
$B = \{$outcomes whose product is 12$\}$
$= \{26, 34, 43, 62\}$
$C = \{$outcomes where the dice differ by 3$\}$
$= \{14, 25, 36, 63, 52, 41\}$.

5) For any event E and sample space S, we define

$$E^c = \{x \in S : x \notin E\}.$$

↑ read "E complement," also denoted $S-E$.

So E^c is the event that E doesn't happen.

Example: for the sets F, G, H above, we have

$F^c = \{$outcomes with at least two heads$\}$
$= \{HHT, THH, HTH, HHH\}$,
$G^c = \{$outcomes where the middle coin is heads$\}$
$= \{TTT, TTH, HTT, HTH\}$
$H^c = \{HTT, THT, TTH, TTT, HTH\}$.

6) For any events D and E, we define

$D \cup E = \{$outcomes in D or E (or both)$\}$
↑ read "D union E" or "D or E"

Example: for the sets A, B, C above,

$A \cup B = \{$outcomes whose sum is 7 or whose product is 12$\}$
$= \{16, 25, 26, 34, 43, 52, 61, 6\}$,
$A \cup C = \{14, 16, 25, 26, 34, 36, 43, 52, 61, 62\}$,
$B \cup C = \{14, 25, 26, 34, 36, 43, 41, 52, 62, 63\}$.

7) For events D and E, we define

$DE = \{$outcomes in both D and $E\}$.
↑ read "D intersect E" or "D and E,"
also written $D \cap E$.

Example: for the sets F, G, H above:

$FG = \{$outcomes with at most one heads where the middle coin is tails$\}$
$= \{HTT, TTH, TTT\}$,
$FH = \emptyset$ (the empty set, also denoted $\{\,\}$)

$GH = \emptyset$.

We can make more complex combinations.
For example:

$A \cup B \cup C$ = {outcomes that sum to 7, multiply to 12, or differ by 3}
$= \{14, 16, 25, 26, 34, 36, 41, 43, 52, 61, 62, 63\}$,

$ABC = \emptyset$,

$F^c \cup G^c$
= {outcomes with at least two heads or where the middle coin is heads}
$= \{HHT, HTH, THH, HHH, THT\}$,

$(F \cup G)^c$ = {outcomes with neither at most one heads nor a tails in the middle}
$= \{HHH, HHT, THH\}$,

$A \cup B^c C$ = {outcomes whose sum is 7, <u>or</u> whose product isn't 12 and whose difference is 3}
$= \{14, 16, 25, 34, 36, 41, 43, 52, 61\}$