FINAL EXAM: SOME PRACTICE PROBLEMS

1. (a) Use the Euclidean Algorithm to find $\gcd(123, 321) = 3$.

   (b) Find natural numbers x and y solving

   $$123x - 321y = \gcd(123, 321).$$

   $$123 \cdot 47 - 321 \cdot 18 = 3.$$

2. (a) Use the Euclidean Algorithm to find $\gcd(247, 156) = 13$.

   (b) Find integers x and y solving

   $$247x - 156y = \gcd(247, 156).$$

   Here, x and y don't need to be positive.

   $$247 \cdot (-5) - 156 \cdot (-8) = \gcd(247, 156).$$

   (c) Find natural numbers x and y solving

   $$247x - 156y = \gcd(247, 156).$$

   $$247 \cdot (-5 + 156) - 156 \cdot (-8 + 247) = \gcd(247, 156).$$
   $$247 \cdot 151 - 156 \cdot 239 = \gcd(247, 156).$$

3. (a) Find natural numbers x and y such that

   $$45x - 56y = 1.$$

   $$45 \cdot 5 - 56 \cdot 4 = 1.$$

   (b) Using the numerization key

A	B	C	D	E	F	G	H	I	J	K	L	M
11	12	13	14	15	16	17	18	19	20	21	22	23
N	O	P	Q	R	S	T	U	V	W	X	Y	Z
24	25	26	27	28	29	30	31	32	33	34	35	36

and the RSA decoding algorithm, with $k = 45$ and $m = 87$, decode the message "17," to obtain a one-letter message. G

4. (a) Find positive integers x and y such hat

$$55x - 64y = 1.$$

$$55 \cdot 7 - 64 \cdot 6 = 1.$$

(b) Using the numerization key above and the RSA decoding algorithm, with $k = 55$ and $m = 85$, decode the message "25," to obtain a one-letter message. E

5. (a) Use the Euclidean algorithm to find $\gcd(31, \varphi(55))$.

Answer: $\gcd(31, \varphi(55)) = 1$

(b) Use the Euclidean algorithm to find integers x and y with $31x - \varphi(55)y = 1$. Here, x and y do not need to be positive.

Answer: $x = -9$, $y = -7$

(c) Tweak your answer to the previous part of this problem, to find *positive* integers (that is, natural numbers) x and y with $31x - \varphi(55)y = 1$.

Answer: $x = 31$, $y = 24$

(d) Using $k = 31$ and $m = 55$, decode the message 12, and denumerize to obtain a single-letter message.

Answer: Message = M