EXAM 2: MORE RANDOM PRACTICE PROBLEMS

1. Use the principle of mathematical induction to prove that, for any $n \in \mathbb{N}$,

$$1 \cdot 3 + 2 \cdot 4 + 3 \cdot 5 + 4 \cdot 6 + \cdots + n(n+2) = \frac{n(n+1)(2n+7)}{6}.$$

2. Identify each of the statements as true or false, by putting a "T" or "F" in the space to the *left* of the statement. Then, in the space to the *right* of the statement, put the *number* of the statement that is the *negation* of the statement in question. For example, if the negation of statement 2 is statement 7, then put a "7" in the space to the right of statement 2.

One of the statements has no negation present, so leave the space to the right of that statement blank.

(Recall that $\mathbb{R}^+$ denotes the set of positive real numbers.)

1. _____ $\forall w \in \mathbb{R}^+, \forall x \in \mathbb{R}, \forall y \in \mathbb{R}, \forall z \in \mathbb{R}^+, w + z \leq x - y$ _____
2. _____ $\exists w \in \mathbb{R}^+, \exists x \in \mathbb{R}, \forall y \in \mathbb{R}, \forall z \in \mathbb{R}^+, w + z \leq x - y$ _____
3. _____ $\exists w \in \mathbb{R}^+, \exists x \in \mathbb{R}, \exists y \in \mathbb{R}, \exists z \in \mathbb{R}^+, x < w + y + z$ _____
4. _____ $\sim(\sim(\forall w \in \mathbb{R}^+, \exists x \in \mathbb{R}, \forall y \in \mathbb{R}, \exists z \in \mathbb{R}^+, x - y < w + z))$ _____
5. _____ $\exists w \in \mathbb{R}^+, \forall x \in \mathbb{R}, \exists y \in \mathbb{R}, \forall z \in \mathbb{R}^+, w + z \leq x - y$ _____
6. _____ $\forall w \in \mathbb{R}^+, \exists x \in \mathbb{R}, \forall y \in \mathbb{R}, \exists z \in \mathbb{R}^+, w + z \leq x - y$ _____
7. _____ $\forall w \in \mathbb{R}^+, \forall x \in \mathbb{R}, \sim(\forall y \in \mathbb{R}, \forall z \in \mathbb{R}^+, w + z \leq x - y)$ _____
8. _____ $\sim(\forall w \in \mathbb{R}^+, \forall x \in \mathbb{R}, \sim(\forall y \in \mathbb{R}, \forall z \in \mathbb{R}^+, x - y < w + z))$ _____
9. _____ $\forall w \in \mathbb{R}^+, \forall x \in \mathbb{R}, \exists y \in \mathbb{R}, \exists z \in \mathbb{R}^+, w + z \leq x - y$ _____

3. (Harder problem.) You wash 14 pairs of socks, each pair a different color from the other pairs, and realize, after taking everything out of the dryer, that 8 socks are lost. So you are left with $(14 \cdot 2) - 8 = 20$ socks. Find the probability that:

A. You are left with 10 matching pairs (this is the best case scenario);
B. You are left with 6 matching pairs (this is the worst case scenario).

Some hints: (i) First, what is the total number of ways of losing 8 out of 28 socks? (ii) To be left with 10 matching pairs is to say that 4 of the original 14 pairs were lost. (iii) To be left with 6 matching pairs is to say that 8 socks, all of different colors, were lost.

How many ways are there of choosing the 8 colors lost, and for each of these colors, how many ways are there of choosing a sock of that color? (iv) The *probability* of scenario A is the number of ways this scenario can occur, divided by the total number of ways of losing 8 out of 28 socks. Similarly for scenario B.