EXAM 1: MORE COMPLETELY RANDOM PRACTICE PROBLEMS

1. Consider sets A, B, and D defined by

$$A = \{\text{even integers}\} = 2\mathbb{Z}, \qquad B = \{3,4,5,6,7,8\}, \qquad D = \{5,6,7,8,9,10\}.$$

Find:

(a) $B - D = \{3,4\}$

(b) $(B \cup D) - A = \{3,5,7,9\}$

(c) $(B \cap D) - A = \{5,7\}$

(d) $\mathscr{P}((B \cap D) - A) = \{\emptyset, \{5\}, \{7\}, \{5,7\}\}$

2. (Note: for this problem, it might help to draw some pictures.) For each $n \in \mathbb{N}$, define a set A_n by

$$A_n = [n+1, n+4] = \{x \in \mathbb{R} : n+1 \leq x \leq n+4\}.$$

Find:

(a) $\bigcup_{n=1}^{3} A_n = [2,7]$

(b) $\bigcap_{n=1}^{3} A_n = [4,5]$

(c) $\bigcup_{n=1}^{\infty} A_n = [2,\infty)$

(d) $\bigcap_{n=1}^{\infty} A_n = \emptyset$

3. (13 points; one point for each blank) Fill in the blanks (there are 13 of them) to complete the proof of the following theorem:

Theorem. For any sets A, B, and C, we have

$$A \subseteq B \Rightarrow C - B \subseteq C - A.$$

Proof. Let A, B, and C be ___sets___.

Assume $A \subseteq$ ___B___. We wish to conclude that $C - B \subseteq$ ___$C - A$___. To do this, assume $x \in$ ___$C - B$___. Then $x \in C$ and $x \notin B$, by definition of ___set difference___.

Now the assumption $A \subseteq B$ is equivalent to the statement $x \in A \Rightarrow$ ___$x \in B$___, which is equivalent to the contrapositive statement $x \notin B \Rightarrow$ ___$x \notin A$___. So, since $A \subseteq B$ and $x \notin B$, we conclude that ___$x \notin A$___. Therefore, since $x \in C$ as already noted, we have $x \in$ ___$C - A$___, by definition of ___set difference___.

We have shown that, if $A \subseteq B$, then $x \in C - B \Rightarrow$ ___$x \in C - A$___. In other words,

$$A \subseteq B \Rightarrow C - B \subseteq \underline{\quad C - A \quad},$$

and we're done. **ATWMR**

(In the last blank above, supply an end-of-proof tagline of your own devising.)

4. Let $A = \{4, 5\}$, $B = \{2, 3\}$, and $C = \{1, 2, 3\}$.

(a) (5 points) Is it true that $C - B \subseteq C - A$? Explain.

Yes it's true, because $C - B = \{1\}$ and $C - A = \{1, 2, 3\}$, and $\{1\} \subseteq \{1, 2, 3\}$.

(b) (6 points) Is it true that $A \subseteq B$?

No, $\{4, 5\} \not\subseteq \{2, 3\}$.

(c) (6 points) Is it true that, for any sets A, B, and C,

$$C - B \subseteq C - A \Rightarrow A \subseteq B?$$

Please explain. (You may want to use parts (a) and (b) of this problem.)

No, this is not true for any sets A, B, and C. Counterexample: for the sets A, B, and C above, we have $C - B \subseteq C - A$, but it's false that $A \subseteq B$.