## EXAM 1: MORE COMPLETELY RANDOM PRACTICE PROBLEMS

**1.** Consider sets $A$, $B$, and $D$ defined by

$$A = \{\text{even integers}\} = 2\mathbb{Z}, \qquad B = \{3,4,5,6,7,8\}, \qquad D = \{5,6,7,8,9,10\}.$$

Find:

(a) $B - D$

(b) $(B \cup D) - A$

(c) $(B \cap D) - A$

(d) $\mathscr{P}((B \cap D) - A)$

**2.** (Note: for this problem, it might help to draw some pictures.) For each $n \in \mathbb{N}$, define a set $A_n$ by

$$A_n = [n+1, n+4] = \{x \in \mathbb{R} : n+1 \leq x \leq n+4\}.$$

Find:

(a) $\displaystyle\bigcup_{n=1}^{3} A_n$

(b) $\displaystyle\bigcap_{n=1}^{3} A_n$

(c) $\displaystyle\bigcup_{n=1}^{\infty} A_n$

(d) $\displaystyle\bigcap_{n=1}^{\infty} A_n$

**3.** (13 points; one point for each blank) Fill in the blanks (there are 13 of them) to complete the proof of the following theorem:

**Theorem.** For any sets $A, B$, and $C$, we have

$$A \subseteq B \Rightarrow C - B \subseteq C - A.$$

**Proof.** Let $A, B$, and $C$ be ____________.

Assume $A \subseteq$ __________. We wish to conclude that $C - B \subseteq$ ______________. To do this, assume $x \in$ ______________. Then $x \in C$ and $x \notin B$, by definition of ____________________.

Now the assumption $A \subseteq B$ is equivalent to the statement $x \in A \Rightarrow$ ______________, which is equivalent to the contrapositive statement $x \notin B \Rightarrow$ ______________. So, since $A \subseteq B$ and $x \notin B$, we conclude that ______________. Therefore, since $x \in C$ as already noted, we have $x \in$ ______________, by definition of ____________________.

We have shown that, if $A \subseteq B$, then $x \in C - B \Rightarrow$ __________________. In other words,

$$A \subseteq B \Rightarrow C - B \subseteq \underline{\hspace{3cm}},$$

and we're done. ____________________

(In the last blank above, supply an end-of-proof tagline of your own devising.)

**4.** Let $A = \{4, 5\}$, $B = \{2, 3\}$, and $C = \{1, 2, 3\}$.

(a) (5 points) Is it true that $C - B \subseteq C - A$? Explain.

(b) (6 points) Is it true that $A \subseteq B$?

(c) (6 points) Is it true that, for any sets $A, B$, and $C$,

$$C - B \subseteq C - A \Rightarrow A \subseteq B?$$

Please explain. (You may want to use parts (a) and (b) of this problem.)