EXAM 1: COMPLETELY RANDOM PRACTICE PROBLEMS

1. Using *only* set facts and definitions and proof strategies from your Exam 1 fact sheet, prove that, if A, B, and C are sets, and $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

2. Prove that $(1 + 3\mathbb{Z}) \cap (1 + 2\mathbb{Z}) = 1 + 6\mathbb{Z}$. You may use the fact that a product of odd numbers is odd.

3. Let $A = \{1, 2, 3\}$ and $B = \{2, 3, 4\}$. Find $\mathscr{P}(A) \cap \mathscr{P}(B)$.

4. Let $A = \{2, \{2\}, \{2, 3\}, 3\}$.

(a) Find $\mathscr{P}(A)$.

(b) Find $A \cap \mathscr{P}(A)$.

5. Given sets A, B, and C, draw a Venn diagram depicting:

(a) $(A \cup B) \cap (A - C)$

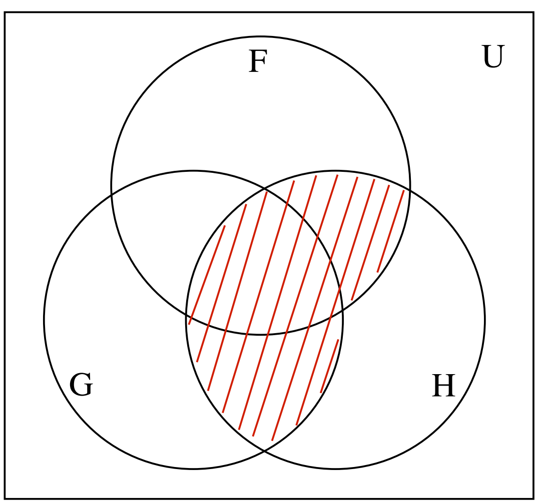
(b) $(A \cap B) \cup (A - C)$

(c) $\big(B - A\big) \cup \big(C - \overline{A}\,\big)$

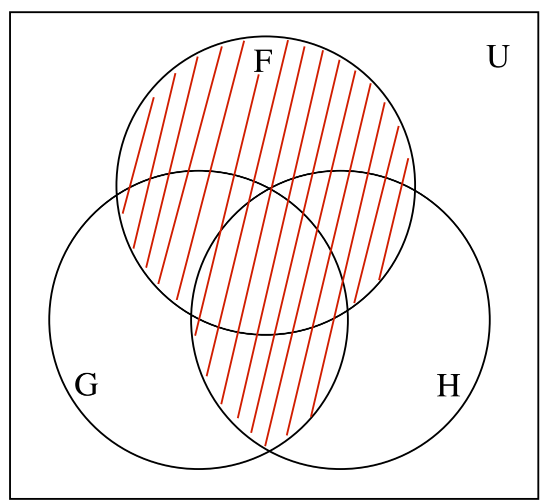
(d) $\big(B - A\big) \cap \big(C - \overline{A}\,\big)$

6. (a) Shade in the indicated set for each of the Venn diagrams below.

$(F \cup G) \cap H$



$F \cup (G \cap H)$



(b) Based on your shadings, what relation do you see between the set you shaded in on the left and the one you shaded in on the right? Your answer should involve the sets F, G, and H, and perhaps things like unions, subsets, intersections, etc.

(c) Prove the relation that you described in part (b) of this problem.

7. For each $n \in \mathbb{N}$, let A_n be the open interval $\left(\frac{1}{n+1}, 2 - \frac{1}{n+1}\right)$. Find:

(a) $\bigcup_{n=1}^{4} A_n$

(b) $\bigcap_{n=1}^{4} A_n$

(c) $\bigcup_{n=1}^{\infty} A_n$

(d) $\bigcap_{n=1}^{\infty} A_n$