

MATH 2001: Intro to Discrete Math

February 27, 2024

Make-up In-class Midterm Exam

I have neither given nor received unauthorized assistance on this exam.

Name: _____

Signature: _____

Please show all work.

Please write neatly. If it's unreadable, it's ungradeable.

If you get stuck on a problem, move on, and then come back to it.

Take a deep breath. Good luck!

DO NOT WRITE IN THIS BOX!

Problem	Points	Score
1	28 pts	
2	24 pts	
3	18 pts	
4	13 pts	
5	17 pts	
TOTAL	100 pts	

1. (28 points; 7 points each)

Consider sets A , B , and D defined by

$$A = \{1, 2, 3, 4, 5, 6, 7, 8, 10\}, \quad B = \{\text{odd integers}\} = 1 + 2\mathbb{Z}, \quad D = \{5, 6, 7, 8, 9, 12\}.$$

Find:

(a) $D - B$

(b) $(A \cup D) - B$

(continued on the next page)

(continued from the previous page)

(c) $(A \cap D) - B$

(d) $\mathcal{P}((A \cap D) - B)$

2. (24 points; 6 points each) (Note: for this problem, it might help to draw some pictures.) For each $n \in \mathbb{N}$, define a set A_n by

$$A_n = \left[1 + \frac{1}{n}, 3\right] = \left\{x \in \mathbb{R} : 1 + \frac{1}{n} \leq x \leq 3\right\}.$$

Find:

(a) $\bigcup_{n=1}^3 A_n$

(b) $\bigcap_{n=1}^3 A_n$

(continued on the next page)

(continued from the previous page)

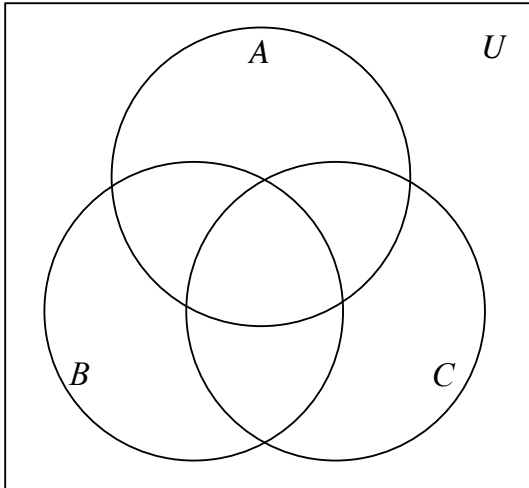
(c) $\bigcup_{n=1}^{\infty} A_n$

(d) $\bigcap_{n=1}^{\infty} A_n$

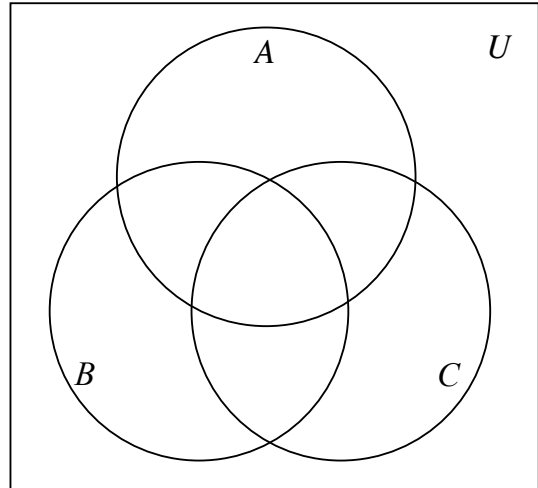
3. (18 points; 9 points each)

(a) Shade in the indicated set for each of the Venn diagrams below.

$$(A \cup B) - C$$



$$A - (B \cup C)$$



(b) Based on your shadings, what relation do you see between the set you shaded in on the left and the one you shaded in on the right? Your answer should involve the sets A , B , and C , and perhaps things like unions, subsets, intersections, etc. You don't have to prove anything. Just state the relation that your answer to part (a) of this problem illustrates.

4. (13 points; one point for each blank) Fill in the blanks (there are 13 of them) to complete the proof of the following theorem:

Theorem. For any sets A , B , and C , we have

$$A \subseteq B \Rightarrow C - B \subseteq C - A.$$

Proof. Let A , B , and _____ be sets.

Assume _____ $\subseteq B$. We wish to conclude that _____ $\subseteq C - A$. To do this, assume $x \in$ _____. Then $x \in C$ and $x \notin$ _____, by definition of set difference.

Now the assumption $A \subseteq B$ is equivalent to the statement _____ $\Rightarrow x \in B$, which is equivalent to the contrapositive statement _____ $\Rightarrow x \notin A$. So, since $A \subseteq B$ and $x \notin$ _____, we conclude that $x \notin A$. Therefore, since $x \in C$ as already noted, we have $x \in$ _____, by definition of set difference.

We have shown that, if $A \subseteq$ _____, then $x \in$ _____ $\Rightarrow x \in C - A$. In other words,

$$A \subseteq B \Rightarrow \text{_____} \subseteq C - A,$$

and we're done. _____

(In the last blank above, supply an end-of-proof tagline of your own devising.)

5. (17 points total) Let $A = \{3, 4, 5, 11, 12\}$, $B = \{5, 6, 12, 83\}$, and $C = \{5, 6, 7\}$.

(a) (5 points) Is it true that $C - B \subseteq C - A$? Explain.

(b) (6 points) Is it true that $A \subseteq B$? Explain.

(c) (6 points) Is it true that, for any sets A, B , and C ,

$$C - B \subseteq C - A \Rightarrow A \subseteq B?$$

Please explain. (You may want to use parts (a) and (b) of this problem.)