1. Recall that the addition principle (AP), together with the Inclusion-Exclusion Principle (IEP), say: to count the number of things with Property 1 or Property 2, or both, add the counts of each, but then you've counted the number that have *both* properties twice, so to compensate, subtract that number once.

   Let's interpret this in terms of *sets*. Fill in the blanks; each blank should involve only of the symbols A, B, $\cup$, $\cap$, and $|\ |$ (that is, the vertical bars used to denote the number of elements in a set), perhaps in combination with each other.

   Let A be the set of items with Property 1, and let B be the set of items with Property 2. Then the set of items with one property or the other (or both) is the set ____ $A \cup B$ ____. To count the elements in this set, we count the number of elements of A, add to this the number of elements of ____ B ____, and then, to compensate for overcounting, we subtract the number of elements in ____ $A \cap B$ ____. In other words, we have the formula

$$|A \cup B| = |A| + \underline{\quad |B| \quad} - \underline{\quad |A \cap B| \quad}.$$

2. Let's generalize the above to three sets. Fill in the blanks with the symbols A, B, C, $\cup$, $\cap$, and $|\ |$, perhaps in combination with each other.

   Let A be the set of items with property 1; let B be the set of items with Property 2; let C be the set of items with Property 3. Then the set of items with at least one of the three properties is the set ____ $A \cup B \cup C$ ____. To count the elements in this set, we count the number of elements of A, add to this the number of elements of ____ B ____, and add to this the number of elements of ____ C ____. But we've overcompensated: we've counted elements of $A \cap B$, and of ____ $A \cap C$ ____, and of ____ $B \cap C$ ____, twice. To compensate, we subtract the number of elements in each of these sets.

   But wait, we're not done! By adding $|A|$ to ____ $|B|$ ____ to ____ $|C|$ ____, we've counted the elements in $A \cap B \cap C$ ____ three ____ times. Moreover, in subtracting $|A \cap B|$ and ____ $|A \cap C|$ ____ and ____ $|B \cap C|$ ____, we've subtracted off the elements in $A \cap B \cap C$ ____ three ____ times. So the net effect is to count the elements in $A \cap B \cap C$ ____ zero ____ times. But we don't want to count them that many times, we want to count them exactly ____ once ____. So let's put them back!

   In other words, all told, we have the formula

$$
\begin{aligned}
|A \cup B \cup C| = |A| + \underline{\quad |B| \quad} + \underline{\quad |C| \quad} \\
- |A \cap B| - \underline{\quad |A \cap C| \quad} - \underline{\quad |B \cap C| \quad} \\
+ \underline{\quad |A \cap B \cap C| \quad}.
\end{aligned}
$$

3. Use the formula from the previous problem to solve (the last part of) the following problem: in a standard 52-card deck, how many 5-card lists:

   (a) Have an ace as the first card;

   $4 \cdot 51 \cdot 50 \cdot 49 \cdot 48 = 23{,}990{,}400$ (we did this one for you, to get you started);

   (b) Contain no face cards (that is, Jacks, Queens, or Kings);

   $\underline{\quad 40 \cdot 39 \cdot 38 \cdot 37 \cdot 36 = 78{,}960{,}960 \quad}$

   (c) Are all of the same suit;

   $\underline{\quad 52 \cdot 12 \cdot 11 \cdot 10 \cdot 9 = 617{,}760 \quad}$

(d) Have an ace as the first card *and* contain no face cards;

$4 \cdot 39 \cdot 38 \cdot 37 \cdot 36 = 7{,}896{,}096$

(e) Have an ace as the first card *and* are all of the same suit;

$4 \cdot 12 \cdot 11 \cdot 10 \cdot 9 = 47{,}520$

(f) Contain no face cards *and* are all of the same suit;

$40 \cdot 9 \cdot 8 \cdot 7 \cdot 6 = 120{,}960$

(g) Have an ace as the first card *and* contain no face cards *and* are all of the same suit;

$4 \cdot 9 \cdot 8 \cdot 7 \cdot 6 = 12{,}096$

(h) Have an ace as the first card *or* contain no face cards *or* are all of the same suit.

$$4 \cdot 51 \cdot 50 \cdot 49 \cdot 48 + 40 \cdot 39 \cdot 38 \cdot 37 \cdot 36 + 52 \cdot 12 \cdot 11 \cdot 10 \cdot 9$$
$$- 4 \cdot 39 \cdot 38 \cdot 37 \cdot 36 - 4 \cdot 12 \cdot 11 \cdot 10 \cdot 9 - 40 \cdot 9 \cdot 8 \cdot 7 \cdot 6$$
$$+ 4 \cdot 9 \cdot 8 \cdot 7 \cdot 6 = 95{,}516{,}640.$$