# Lists, continued.

Recall: a list is an ordered sequence of items.

Guidelines/principles for counting lists:

1. Multiplication principle (MP):
   (a) If Thing 1 can be done in $m$ ways and, for each of these ways, Thing 2 can be done in $n$ ways, then Things 1 and 2 together can be done in $mn$ ways.

   (b) The number of length-$k$ lists that can be made from $n$ items is
   (i) $n^k$ allowing repetition;
   (ii) $n(n-1)(n-2)\cdots(n-k+1)$* disallowing repetition.

   *Sometimes denoted ${}_nP_k$ or $P(n,k)$.
   Note: a length-$k$ list from $n$ items, without repetition, is called a <u>k-permutation</u> of $n$.

2) (a) Addition principle (AP):
   If Thing 1 can be done in $m$ ways and Thing 2 can be done in $n$ ways, then the number of ways of doing Thing 1 <u>or</u> Thing 2 is $m+n$, <u>provided</u> you're not counting twice.

   (b) Inclusion-Exclusion Principle (IEP):
   If you <u>are</u> counting twice, <u>subtract</u>

to compensate.

3) Subtraction principle (SP):
number of lists with a property $P$
= total number of lists
− number without property $P$.

Example 1.

How many 5-card lists can be dealt from a standard 52-card deck if:

(A) all have the same suit;
(B) Exactly one card is a 3;
(C) No card is a 3;
(D) At least one card is a 3;
(E) All have the same suit or no card is a 3 (or both).

Solution.

(A) The first card can be anything; its suit determines the suit of the other cards. So there are

$$52 \cdot 12 \cdot 11 \cdot 10 \cdot 9 = 617,760 \text{ such lists.}$$

(We've used MP(a).)

(B) $\exists$ $4 \cdot 48 \cdot 47 \cdot 46 \cdot 45$ lists where 3 is the first card, $48 \cdot 4 \cdot 47 \cdot 46 \cdot 45$ where it's the second, etc. Total:

$$5(4 \cdot 48 \cdot 47 \cdot 46 \cdot 45) = 93,398,400 \text{ such lists.}$$

(We've used MP(a) and AP(a).)

(c) 48 cards total are not 3's. So there are
$48 \cdot 47 \cdot 46 \cdot 45 \cdot 44 = 205{,}476{,}480$
lists with no 3's.
(We've used MP(a) or, equivalently, MP(b)(ii).)

(D) $\overbrace{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48}^{\text{all possible 5-card lists}} - \overbrace{48 \cdot 47 \cdot 46 \cdot 45 \cdot 44}^{\text{those with no 3's.}}$
$= 106{,}398{,}720$ such lists, by
MP(a) (or b(ii)) together with SP.

(E) The number of lists where all cards have the same suit <u>and</u> no card is a 3 is

$48 \cdot 11 \cdot 10 \cdot 9 \cdot 8 = 380{,}160$, by MP(a).

So, by the answers to parts (A) and (C) above, and by IEP, the number of lists with all cards the same suit <u>or</u> with no 3's is

$617{,}760 + 205{,}476{,}480 - 380{,}160$

$= 205{,}714{,}080.$

<u>Example 2.</u>
Consider length-4 lists, with no repetition, made from the letters a, b, c, d, e, f, g.

(a) How many such lists have exactly one vowel?
(b) How many have at least one vowel??

Solution.

(a) The number with a vowel in the first place only is

$$2 \cdot 5 \cdot 4 \cdot 3 = 120$$

Similarly, a vowel in the second place only can happen in

$$5 \cdot 2 \cdot 4 \cdot 3 \text{ ways.}$$

So exactly one vowel can happen in

$$4 \cdot (2 \cdot 5 \cdot 4 \cdot 3) = 480 \text{ ways.}$$

(b) The number is

total # of length-4 lists

− # with no vowels

$$= 7 \cdot 6 \cdot 5 \cdot 4 - 5 \cdot 4 \cdot 3 \cdot 2 = 720.$$