In the space below, on this page, list at least five things that are wrong with the following proof. Be as specific as possible.

Then, on the reverse side of this page, supply a correct proof.

Theorem. Let A, B, C, D, and E, be sets. Then

$$C \subseteq D \Rightarrow (A \cap B) \cap (C - E) \subseteq (A \cup B) \cap D.$$

Proof. Let $x \in (A \cap B) \cap (C - E)$. We have $A \cap B \subseteq A$ and $A \subseteq A \cup B$, so $A \cap B \subseteq A \cup B$. So $x \in A \cup B$.

Also, $C - E \subseteq C \subseteq D$, so $x \in D$.

Since $x \in A \cup B$ and $x \in D$, we have $x \in (A \cup B) \cap D$.

So $C \subseteq D \Rightarrow (A \cap B) \cap (C - E) \subseteq (A \cup B) \cap D$.

Here are some problems with the proof:

1) It does not start out with the assumption "Let A, B, C, D, and E, be sets" OR the assumption "Assume $C \subseteq D$."

2) Where it says "We have $A \cap B \subseteq A$ and $A \subseteq A \cup B$, so $A \cap B \subseteq A \cup B$," an assumption is being made that we haven't proved. The assumption is of the form "if $X \subseteq Y$ and $Y \subseteq Z$, then $X \subseteq Z$," which is true but we haven't proved it.

3) We HAVE proved that, for any sets S and T, we have $S \cap T \subseteq S$ and $S \subseteq S \cup T$ (see S-POP, Proposition B(ii)-1_E and Exercise B(ii)-3), but the above proof does not cite these results.

4) Although it's pretty obvious, we have not proved that, for sets C and E, we have $C - E \subseteq C$.

5) We're stating that $C - E \subseteq C \subseteq D$, and concluding (implicitly) that $C \subseteq D$. Again, we're assuming a result of the form 'if $X \subseteq Y$ and $Y \subseteq Z$, then $X \subseteq Z$," which is true but we haven't proved it.

6) Nowhere have we invoked the relevant definitions (like "by definition of union," etc.).

7) There is nothing, like $\square$ for example, to indicate the end of the proof.

Theorem. Let A, B, C, D, and E, be sets. Then

$$C \subseteq D \Rightarrow (A \cap B) \cap (C - E) \subseteq (A \cup B) \cap D.$$

Proof. Let A, B, C, D, and E, be sets. Assume that $C \subseteq D$. We wish to conclude that $(A \cap B) \cap (C - E) \subseteq (A \cup B) \cap D$.

So let $x \in (A \cap B) \cap (C - E)$. Then $x \in A \cap B$ and $x \in C - E$, by definition of intersection.

Since $x \in A \cap B$, we have $x \in A$, by definition of intersection. But then $x \in A \cup B$, by definition of union.

Moreover, since $x \in C - E$, we have $x \in C$ and $x \notin E$, by definition of set difference. So certainly $x \in C$. But then, since $C \subseteq D$, we have $x \in D$.

Since $x \in A \cup B$ and $x \in D$, we have $x \in (A \cup B) \cap D$, by definition of intersection.

So $(A \cap B) \cap (C - E) \subseteq (A \cup B) \cap D$. $\square$