More on sets:

universe, complement, Venn diagrams.

1) In some contexts, we assume that all sets in question are subsets of some universe U. E.g. if we're discussing properties of integers, we might stipulate that $U = \mathbb{Z}$.

2) Given a universe U and a set $A \subseteq U$, we define the <u>complement</u> $\bar{A}$ of A (in U) by
$$\bar{A} = U - A.$$

Examples:

(a) Let $U = \{a, b, c, d, e\}$.
Then
$$\overline{\{a, b\}} = \{c, d, e\},$$
$$\overline{\{c, d, e\}} = \{a, b\}$$
(note that, for any set A and universe U, $\bar{\bar{A}} = A$).

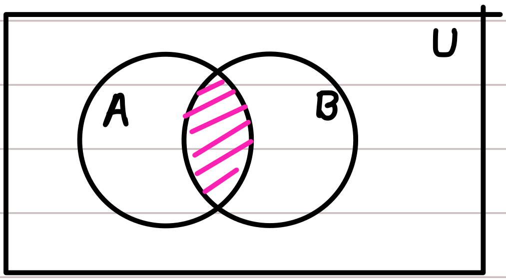
(b) Let $U = \mathbb{Z}$.
Then
$$\overline{\{\text{odd numbers}\}} = \{\text{even numbers}\},$$
$$\overline{3\mathbb{Z}} = (1 + 3\mathbb{Z}) \cup (2 + 3\mathbb{Z}),$$
$$\overline{\mathbb{N}} = \{\ldots, -3, -2, -1, 0\},$$
$$\overline{\varnothing} = \mathbb{Z},$$
$$\overline{\mathbb{Z}} = \varnothing,$$
etc.

3) Venn diagrams.

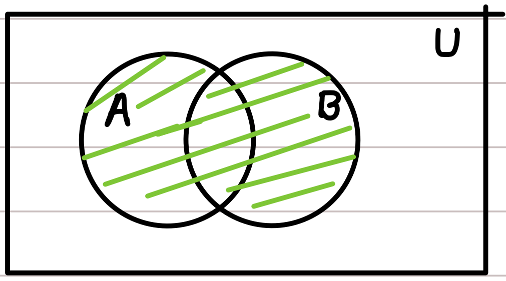
Depict the universe U as a box; sets are regions in the box.

Use Venn diagrams (with shading, if it helps) to:

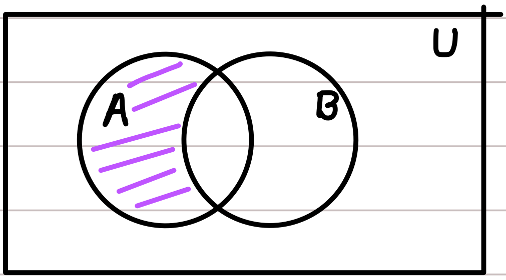
(a) Depict set operations. Examples:



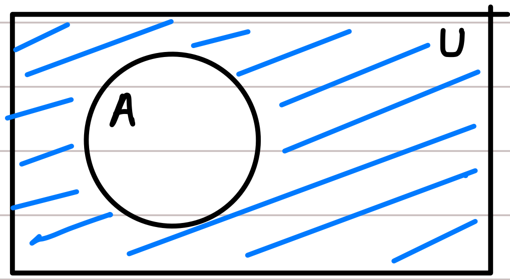
(i) $A \cap B$



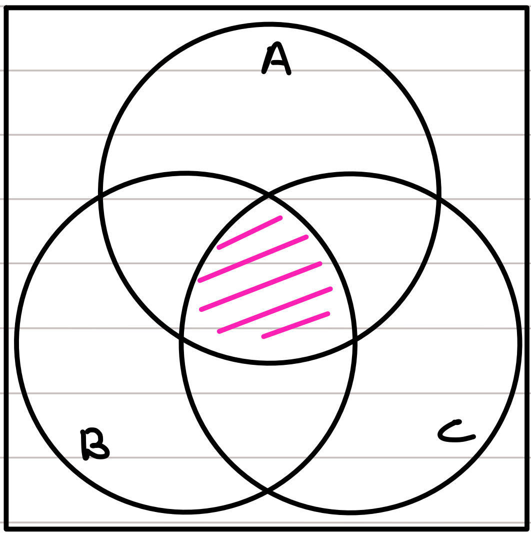
(ii) $A \cup B$



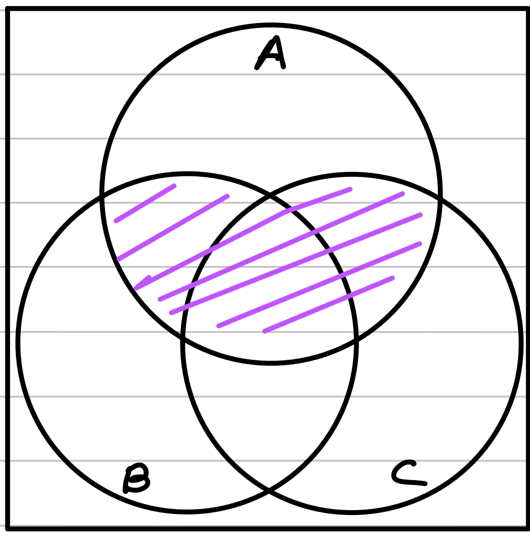
(iii) $A - B$



(iv) $\bar{A}$

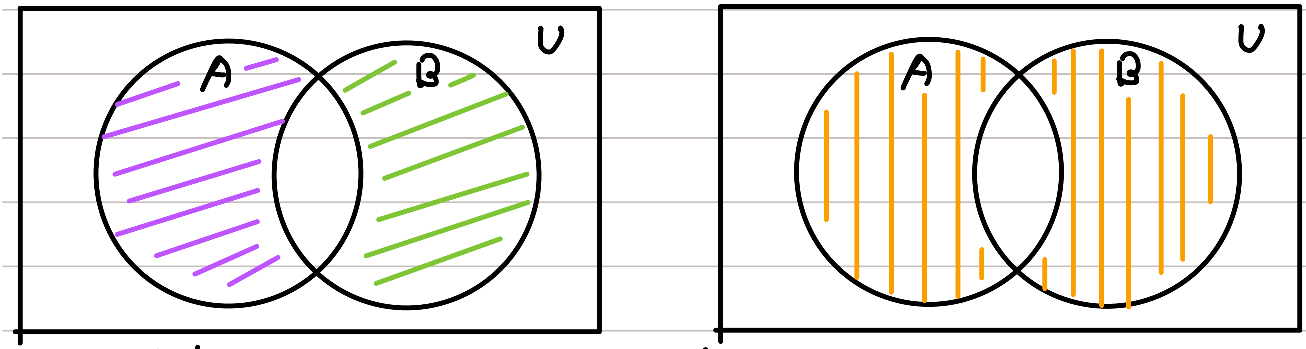


(v) $A \cap B \cap C$



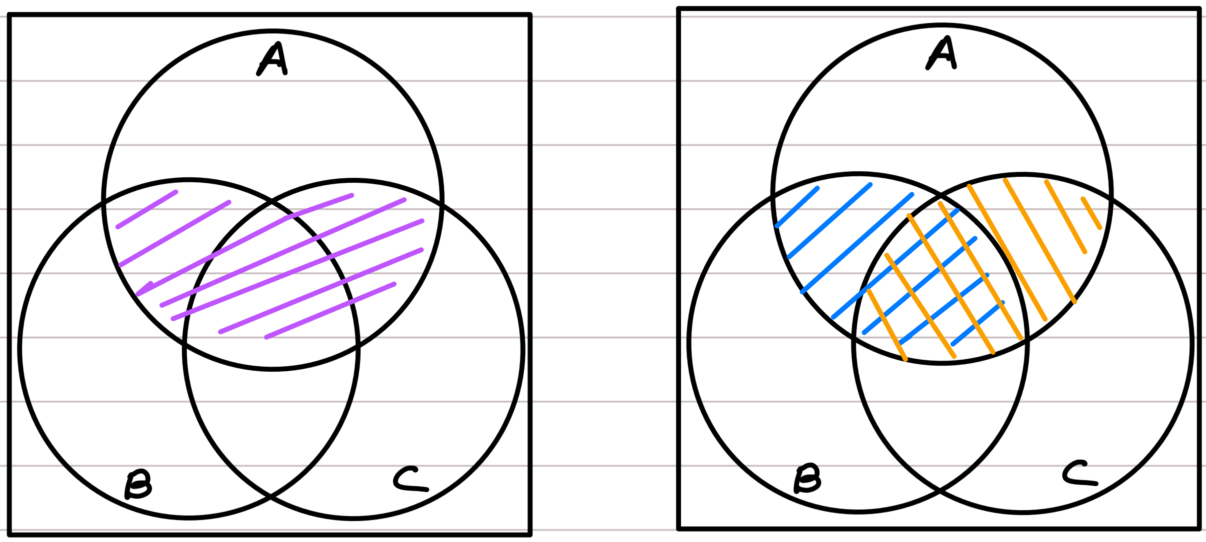
(vi) $A \cap (B \cup C)$

(b) Illustrate set <u>relations</u> (facts). Examples:



(i) Illustrates that

$$(A-B) \cup (B-A) = (A \cup B) - (A \cap B)$$



(ii) Illustrates that

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

(Illustrations are not proofs!)