Sets.

Definition 1.

A <u>set</u> is a collection of objects, called <u>elements</u> of the set.

Ways of describing/defining/denoting sets:

- words
- symbols/names
- listing elements
- "set builder notation".

Examples.

symbol/name

1) L = the set of distinct letters in the word "mathematikvergnugen" — words

$= \{a, e, g, h, i, k, m, n, r, t, u, v\}$
$= \{m, a, t, h, e, i, k, v, r, g, n, u\}$ — listings (order doesn't matter)

$= \{\text{letters } \alpha : \alpha \text{ is a letter in "mathematikvergnugen"}\}$ — set builder notation

Note: the braces mean "the set consisting of;" the colon means "such that."

2) $\mathbb{Z}$ = the set of all integers
$= \{\ldots, -2, -1, 0, 1, 2, \ldots\}$
$= \{0, \pm 1, \pm 2, \ldots\}$

The symbol $\mathbb{Z}$ is <u>reserved</u> for the set of integers.

3) E = the set of even integers $= \{0, \pm 2, \pm 4, \ldots\}$
$= \{2n : n \in \mathbb{Z}\}$

the symbol "$\in$" means "belongs to" or "is an element of"

$= \{n \in \mathbb{Z} : n = 2k \text{ for some } k \in \mathbb{Z}\}.$

4) $F = \{m \in E : -6 \le m \le 4\}$
$= \{-6, -4, -2, 0, 2, 4\}$

5) $2 + 7\mathbb{Z} = \{n \in \mathbb{Z} : n = 2 + 7k \text{ for some } k \in \mathbb{Z}\}$
$= \{\ldots, -12, -5, 2, 9, \ldots\}$

6) In general, for $a, b \in \mathbb{Z}$, $a + b\mathbb{Z}$ denotes $\{n \in \mathbb{Z} : n = a + bk \text{ for some } k \in \mathbb{Z}\}$.
E.g. the set E above may be denoted $0 + 2\mathbb{Z}$, also written $2\mathbb{Z}$. Similarly, {odd integers} may be denoted $1 + 2\mathbb{Z}$.

7) $[-3, 5) = \{\text{real numbers } x : -3 \le x < 5\}$.
$(-3, 5) = \{\text{real numbers } x : -3 < x < 5\}$.
Warning: $(-3, 5)$ also denotes a point in the plane!!

8) $SL(2, \mathbb{Z}) = \{\text{matrices } \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \mathbb{Z},$
$ad - bc = 1\}$.

9) More special, reserved symbols:

$\mathbb{R} = \{\text{real numbers}\}$
$\mathbb{R}^2 = \{\text{ordered pairs } (x, y) : x, y \in \mathbb{R}\}$
$\mathbb{N} = \{\text{natural numbers}\} = \{n \in \mathbb{Z} : n > 0\}$
$\mathbb{Q} = \{\text{rational numbers}\}$
$= \{m/n : m, n \in \mathbb{Z} \text{ and } n \ne 0\}$
$\emptyset$ = <u>the empty set</u> (the set with no elements), also denoted $\{\ \}$.

10) You can have sets of sets, or sets containing sets and other things, like

$$\{\{1,2\},\{3\}\},\ \{\emptyset\},\ \{\{\pi,5,\sqrt{2}\},\ x,y,z\},$$
$$\{7,\{7\},\ \{7,\{7\}\}\}.$$

<u>Definition 2.</u>

Let A, B be sets. We say <u>A is a subset of B</u>, written $A \subseteq B$, if every element of A is also in B (that is: if no element of A lies outside of B). Otherwise, we write $A \nsubseteq B$.

E.g. for the sets defined above:

$\mathbb{N} \subseteq \mathbb{Z}$; $\mathbb{Z} \subseteq \mathbb{R}$ (we can write $\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{R}$);
$2+7\mathbb{Z} \subseteq \mathbb{Z}$; $[-3,5) \subseteq \mathbb{R}$; $F \subseteq E$; $\mathbb{Q} \nsubseteq \mathbb{Z}$; $F \nsubseteq 2+7\mathbb{Z}$;

$\{\{1,2\}\} \subseteq \{\{1,2\},\{3\}\}$,
$\emptyset \subseteq$ any set whatsoever;
any set whatsoever $\subseteq$ itself.

<u>Definition 3</u>

The <u>cardinality</u> of a set S, denoted $|S|$, is the number of elements of S.

E.g. for the sets above,

$|F| = 6$, $|L| = 12$, $|\emptyset| = 0$,

$$|\{\{1,2\},\{3\}\}| = 2,$$

$$|\mathbb{N}| = |\mathbb{Q}| = |\mathbb{R}| = |[-3,5)| = |E| = |2+7\mathbb{Z}| = |SL(2,\mathbb{Z})|$$
$$= \infty.$$