Solutions to Selected Exercises, HW #2

Assignment:

- S-POP, Part B(ii): Exercises B(ii)-1, 2, 3, 4.
- T-BOP, Section 1.2: Exercises A1(abgh), A2(bf), A3, A8.
- T-BOP, Section 1.3: Exercises A1, A6, C13, C15.
- T-BOP, Section 1.4: Exercises A2, A4, A5, B14, B15.
- T-BOP, Section 1.5: Exercises 1, 4ade.

S-POP, Part B(ii)

Exercise B(ii)-1. Show that the set of all integer multiples of 4 is contained in the set of all even numbers.

Proof. Let $A = \{\text{all integer multiples of } 4\} = 4\mathbb{Z}$, and let $B = \{\text{all even numbers}\} = 2\mathbb{Z}$. We wish to show that $A \subseteq B$.

So let $x \in A$. Then, by definition of A, $x = 4k$ for some $k \in \mathbb{Z}$. But $4 = 2 \cdot 2$, so

$$x = (2 \cdot 2)k = 2 \cdot (2k) = 2m,$$

where $m = 2k \in \mathbb{Z}$. So, by definition of B, $x \in B$.

So $A \subseteq B$. $\square$

Exercise B(ii)-3. Show that, for any sets S and T, we have $S \subseteq S \cup T$.

SOLUTION:

Proof. Let S and T be sets. Let $x \in S$. Then certainly $x \in S$ or $x \in T$, so $x \in S \cup T$, by definition of union.

So $S \subseteq S \cup T$. $\square$

T-BOP, Section 1.2

Exercise A2: **(b)** $B \times A = \{(0, \pi), (0, e), (0, 0), (1, \pi), (1, e), (1, 0)\}$.

(f) Technically, $(A \times B) \times B$ is the set of all ordered pairs (x, y), where $x \in A \times B$ and $y \in B$. So technically, $(A \times B) \times B$ looks like this:

$$\begin{aligned}(A \times B) \times B = \{&((\pi,0),0), ((\pi,1),0), ((e,0),0), ((e,1),0), ((0,0),0), ((0,1),0)\big),\\ &((\pi,0),1), ((\pi,1),1), ((e,0),1), ((e,1),1), ((0,0),1), ((0,1),1)\big)\}.\end{aligned}$$

But it's OK to think of $(A \times B) \times B$ as being the same as $A \times B \times B$. So we could write

$$\begin{aligned}(A \times B) \times B = \{&(\pi,0,0), (\pi,1,0), (e,0,0), (e,1,0), (0,0,0), (0,1,0),\\ &(\pi,0,1), (\pi,1,1), (e,0,1), (e,1,1), (0,0,1), (0,1,1)\}.\end{aligned}$$

Do you see the difference? It's subtle, and I'll accept either answer, but there is, technically, a distinction. See the bottom of p. 10 of T-BOP.

Exercise A8.

$$\{0,1\}^4 = \{(0,0,0,0),(0,0,0,1),(0,0,1,0),(0,0,1,1),(0,1,0,0),(0,1,0,1),(0,1,1,0),(0,1,1,1),\\ (1,0,0,0),(1,0,0,1),(1,0,1,0),(1,0,1,1),(1,1,0,0),(1,1,0,1),(1,1,1,0),(1,1,1,1)\}.$$

T-BOP, Section 1.3

Exercise A6. $\emptyset, \{\mathbb{R}\}, \{\mathbb{Q}\}, \{\mathbb{N}\}, \{\mathbb{R},\mathbb{Q}\}, \{\mathbb{Q},\mathbb{N}\}, \{\mathbb{R},\mathbb{N}\}, \{\mathbb{R},\mathbb{Q},\mathbb{N}\}$.

T-BOP, Section 1.4

Exercise A4. $\mathscr{P}(\{\mathbb{R},\mathbb{Q}\}) = \{\emptyset, \{\mathbb{R}\}, \{\mathbb{Q}\}, \{\mathbb{R},\mathbb{Q}\}\}$.

Exercise A14. $\big|\mathscr{P}(\mathscr{P}(A))| = 2^{|\mathscr{P}(A))|} = 2^{2^m}$.

Remark: 2^{2^m} means $2^{(2^m)}$, not $(2^2)^m$. They're not the same; for example, $2^{(2^3)} = 2^8 = 256$, while $(2^2)^3 = 4^3 = 64$.

T-BOP, Section 1.5

Exercise 4: **(a)** $A \times B = \{(b,a),(b,b),(c,a),(c,b),(d,a),(d,b)\}$ and $B \times B = \{(a,a),(a,b),(b,a),(b,b)\}$, so $(A \times B) \cup (B \times B) = \{(b,a),(b,b)\}$.

(d) $(A \cap B) \times A = \{1\} \times \{0,1\} = \{(1,0),(1,1)\}$.

(e) $(A \times B) \cap B = \emptyset$, since $A \times B$ is a set of ordered pairs but B contains no ordered pairs.