In this activity, we are going to consider a curious *conditional probability* scenario. Conditional probability is when you ask questions like: what's the probability of Thing 1 happening, *given* that Thing 2 has happened?

The scenario we will consider here is this. Suppose you roll two fair dice. *Given* that one of the dice lands on a 5, what's the probability that the other one does too?

We'll consider this question from three perspectives: (1) intuitively; (2) empirically (that is, experimentally), and (3) mathematically.

1. Here's one intuitive way of looking at this problem (fill in the blanks): Whether a given die lands on a 5 should have nothing to do with whether the other one does. So the probability of a die landing on 5, given that the other one did, should just be the probability of a die landing on 5, period. This probability is __1/6__ (express as a fraction), or __16.67%__ (express as a percent, to two decimal places), since there are __6__ possible numbers that can come up on the die, and only __one__ of these is a 5.

2. Now let's conduct an experiment.

(a) Do the following:

(i) Roll your two dice.

(ii) If at least one of the dice (that is, if either one or two of the dice) lands on 5, then put a tally mark in the first row of the frequency table on the next page, where it says "Event B: at least one of the dice lands on 5." Otherwise, do nothing.

(ii) If you *did* put a tally mark in the event B row – that is, if at least one of your dice landed on 5 – AND if the other die landed on 5 as well (that is, if both dice landed on 5), then put a tally mark in the second row of the frequency table on the next page, where it says "Event A: both dice land on 5." Otherwise, do nothing.

Now repeat (i,ii,iii) *at least* 100 times!

(over)

Event	Frequency
Event B: (at least) one die lands on 6.	60895
Event A: both dice land on 6.	5548

(b) Now, divide the number of tally marks in the second row (the "Event A" row) by the number of tally marks in the first row (the "Event B" row). What is the number you got?

Answer:

$$\frac{\text{Event } A \text{ tally}}{\text{Event } B \text{ tally}} = \underline{\quad 5548/60895 = 0.0911 = 9.11\% \quad}$$

(express as a fraction, and as a percent to two decimal places).

(c) Explain why the number you got at the end of part (b) above should give you some sort of idea of the probability of a die landing on 5, given that the other one did.

Explanation: We are observing whether one die lands on 5, and only then do we consider whether the other does as well. So it makes sense that dividing the number of double 5's by the number of outcomes with at least one 5 should give us an idea of the probability of two 5's given that there is at least one.

3. Now, we consider the question

> What's the probability of a die landing on 5, given that the other one did?

mathematically. Here's how.

Imagine that the outcome of rolling your two dice is recorded as a string of two numbers (like 42), the first number being what comes up on the green die, and the second being what comes up on the red one.

(a) Write down, explicitly, all outcomes where at least one of the dice lands on a 5. How many of these outcomes are there?

15, 25, 35, 45, 55, 65, 51, 52, 53, 54, 55.

There are 11 outcomes where at least one die is a 5.

(b) Write down, explicitly, all outcomes where both dice land on a 5. How many of these outcomes are there?

55. There's just one such outcome.

(c) Divide your answer to part (b) of this problem by your answer to part (a) of this problem. What do you get? Express your answer as a fraction, and as a percent to two decimal places.

$$\frac{1}{11} = 0.0909 = 9.09\%.$$

(d) Explain why the number you got at the end of part (c) above should give you some sort of idea of the probability of a die landing on 5, given that the other one did.

Dividing the number of outcomes with two 5's by the number of outcomes with at least one 5 should give us an idea of how often two 5's happen, relative to the frequency of at least one 5 happening.

4. Of the above three measures (intuitive, empirical, mathematical) of the probability of a die landing on 5, given that the other one did, which measure do you think is best? Why?

Probably the mathematical measure, because it's math, after all. Also, the mathematical measure agrees best with the empirical measure.