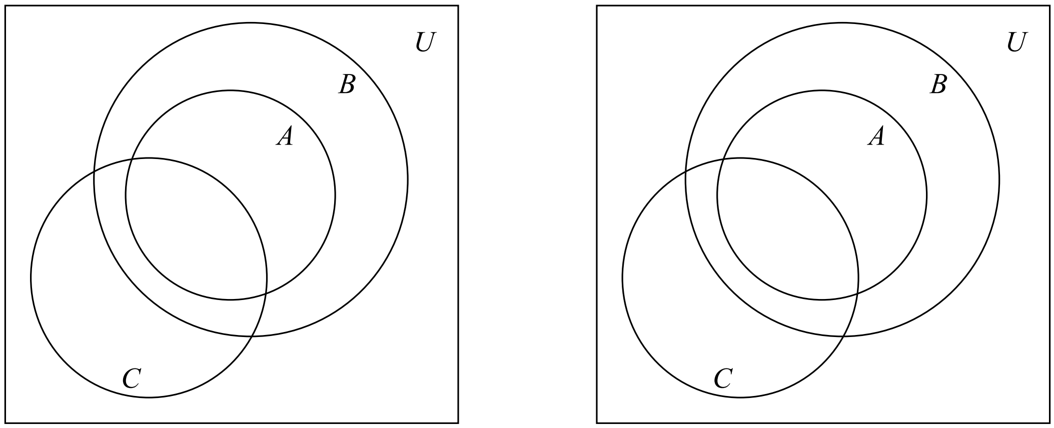
1. Consider the following pair of identical Venn diagrams, each depicting sets A, B, and C, where $A \subseteq B$.



(a) In the diagram on the left, shade in the set $C - B$.

(b) In the diagram on the right, shade in the set $C - A$.

(c) Fill in the blank to complete the following theorem, illustrated by the above Venn diagram:

Theorem. For any sets A, B, and C, we have

$$A \subseteq B \Rightarrow \underline{\quad C - B \subseteq C - A \quad}.$$

(d) Fill in the blanks to complete the proof of the above theorem.

Proof. Let A, B, and C be __sets__.

Assume $A \subseteq$ __B__. We wish to conclude that __$C - B \subseteq C - A$__. To do this, assume $x \in$ __$C - B$__. Then $x \in C$ and $x \notin B$, by definition of __set difference__. Now the assumption $A \subseteq B$ is equivalent to the statement $x \in A \Rightarrow$ __$x \in B$__, which is equivalent to the contrapositive statement __$x \notin B \Rightarrow x \notin A$__. So, since $A \subseteq B$ and $x \notin B$, we conclude that __$x \notin A$__. Therefore, since $x \in C$ as already noted, we have $x \in$ __$C - A$__, by definition of __set difference__.

We have shown that, if $A \subseteq B$, then $x \in C - B \Rightarrow$ __$x \in C - A$__. In other words,

$$A \subseteq B \Rightarrow \underline{\quad C - B \subseteq C - A \quad},$$

and we're done. **ATWMR**

(In the last blank above, supply an end-of-proof tagline devised by your group.)

2. Given sets A, B, and C, is it always true that

$$(C \cup B) - A = C \cup (B - A)?$$

If so, prove it. If not, give a counterexample.

SOLUTION. This is false. For example, let

$$A = \{1, 2, 3, 4, 5\}, \qquad B = \{3, 4, 5, 6\} \qquad C = \{1, 2, 10, \pi, 73\}.$$

Then $C \cup B = \{1, 2, 3, 4, 5, 6, 10, \pi, 73\}$, so

$$(C \cup B) - A = \{6, 10, \pi, 73\},$$

while $B - A = \{6\}$, so

$$C \cup (B - A) = \{1, 2, 6, 10, \pi, 73\}.$$

Note that, although these sets aren't equal, it is true, in this case, that

$$(C \cup B) - A \subseteq C \cup (B - A).$$

In fact, this is always true, as you can convince yourself with a quick proof.