MATH 2001: Intro to Discrete Math

February 21, 2024

In-class Midterm Exam

I have neither given nor received unauthorized assistance on this exam.

Name: SOLUTIONS

Signature:____________________

Please show all work.

Please write neatly. If it's unreadable, it's ungradeable.

If you get stuck on a problem, move on, and then come back to it.

Take a deep breath. Good luck!

DO NOT WRITE IN THIS BOX!

Problem	Points	Score
1	28 pts	
2	24 pts	
3	18 pts	
4	13 pts	
5	17 pts	
TOTAL	100 pts	

1. (28 points; 7 points each)

Consider sets A, B, and D defined by

$$A = \{\text{even integers}\} = 2\mathbb{Z}, \qquad B = \{3, 4, 5, 6, 7, 8\}, \qquad D = \{5, 6, 7, 8, 9, 10\}.$$

Find:

(a) $B - D = \{3, 4\}$

(b) $(B \cup D) - A = \{3, 5, 7, 9\}$

(c) $(B \cap D) - A = \{5, 7\}$

(d) $\mathscr{P}((B \cap D) - A) = \{\emptyset, \{5\}, \{7\}, \{5, 7\}\}$

2. (24 points; 6 points each) (Note: for this problem, it might help to draw some pictures.) For each $n \in \mathbb{N}$, define a set A_n by

$$A_n = [n + 1, n + 4] = \{x \in \mathbb{R} : n + 1 \leq x \leq n + 4\}.$$

Find:

(a) $\bigcup_{n=1}^{3} A_n = [2, 7]$

(b) $\bigcap_{n=1}^{3} A_n = [4, 5]$

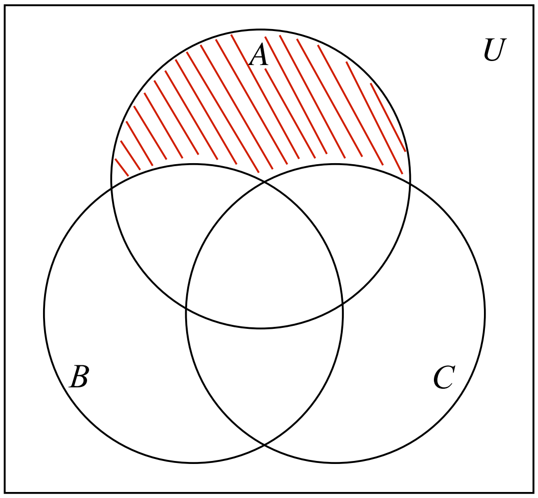
(c) $\bigcup_{n=1}^{\infty} A_n = [2, \infty)$

(d) $\bigcap_{n=1}^{\infty} A_n = \emptyset$

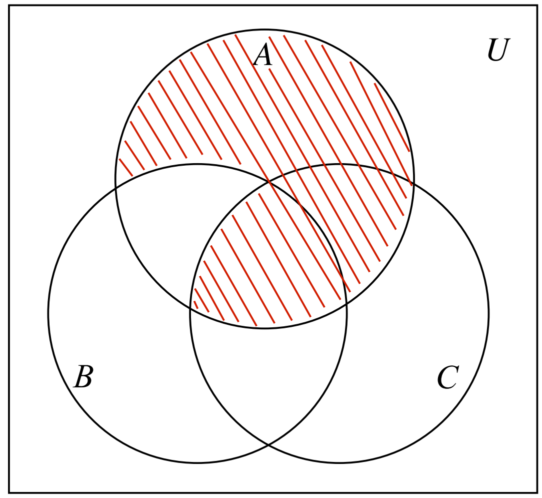
3. (18 points; 9 points each)

(a) Shade in the indicated set for each of the Venn diagrams below.

$(A - B) - C$



$A - (B - C)$



(b) Based on your shadings, what relation do you see between the set you shaded in on the left and the one you shaded in on the right? Your answer should involve the sets A, B, and C, and perhaps things like unions, subsets, intersections, etc. You don't have to prove anything. Just state the relation that your answer to part (a) of this problem illustrates.

$(A - B) - C \subseteq A - (B - C)$.

4. (13 points; one point for each blank) Fill in the blanks (there are 13 of them) to complete the proof of the following theorem:

Theorem. For any sets A, B, and C, we have

$$A \subseteq B \Rightarrow C - B \subseteq C - A.$$

Proof. Let A, B, and C be ___sets___.

Assume $A \subseteq$ ___B___. We wish to conclude that $C - B \subseteq$ ___$C - A$___. To do this, assume $x \in$ ___$C - B$___. Then $x \in C$ and $x \notin B$, by definition of ___set difference___.

Now the assumption $A \subseteq B$ is equivalent to the statement $x \in A \Rightarrow$ ___$x \in B$___, which is equivalent to the contrapositive statement $x \notin B \Rightarrow$ ___$x \notin A$___. So, since $A \subseteq B$ and $x \notin B$, we conclude that ___$x \notin A$___. Therefore, since $x \in C$ as already noted, we have $x \in$ ___$C - A$___, by definition of ___set difference___.

We have shown that, if $A \subseteq B$, then $x \in C - B \Rightarrow$ ___$x \in C - A$___. In other words,

$$A \subseteq B \Rightarrow C - B \subseteq \underline{\quad C - A \quad},$$

and we're done. ___**ATWMR**___

(In the last blank above, supply an end-of-proof tagline of your own devising.)

5. (17 points total) Let $A = \{4, 5\}$, $B = \{2, 3\}$, and $C = \{1, 2, 3\}$.

(a) (5 points) Is it true that $C - B \subseteq C - A$? Explain.

Yes it's true, because $C - B = \{1\}$ and $C - A = \{1, 2, 3\}$, and $\{1\} \subseteq \{1, 2, 3\}$.

(b) (6 points) Is it true that $A \subseteq B$?

No, $\{4, 5\} \not\subseteq \{2, 3\}$.

(c) (6 points) Is it true that, for any sets A, B, and C,

$$C - B \subseteq C - A \Rightarrow A \subseteq B?$$

Please explain. (You may want to use parts (a) and (b) of this problem.)

No, this is not true for any sets A, B, and C. Counterexample: for the sets A, B, and C above, we have $C - B \subseteq C - A$, but it's false that $A \subseteq B$.