Quick intro to probability.

Six definitions and an axiom (with examples):

(i) <u>Experiment</u>: a repeatable procedure (e.g.: flip a coin; flip 3 coins; roll two dice; draw 5 cards).

(ii) <u>Sample space</u> S: the set of possible outcomes of an experiment. E.g. roll two dice: we might write

$$S = \{11, 12, \ldots, 16, 21, 22, \ldots, 26, 31, \ldots, 66\}.$$

Here $|S| = 6^2 = 36$.

(iii) <u>Event</u> A: a subset of the sample space. E.g. for S as above, the event

$A = \{13, 31, 22\}$ is the event "the sum on the dice is 4."

(iv) $P(A)$: the <u>probability</u> of the event A.

<u>Probability Axiom</u> #1: if all outcomes of an experiment are equally likely, then

$$P(A) = \frac{|A|}{|S|}$$ (assuming $|A|$ and $|S|$ are finite).

E.g. roll two fair dice. Then for S and A as above,

$$P(A) = \frac{3}{36} \approx 0.083 = 8.33\ \%.$$

(v) <u>Random Variable</u> (rv) X: a way to assign a <u>number</u> to each outcome of an experiment.

E.g. if we roll two fair dice, we might define

X = the number of 5's that come up.
(So $X = 0, 1,$ or 2).

Other rv's for this same experiment:

Y = the sum of the two #'s,
Q = the smallest of these #'s,
Z = the largest,
Λ = the average.

(vi) Probability mass function (pmf) of an rv X: specification of the probability $P(X=x)$ (meaning the probability that X takes the value x), for each possible value x.

Example 1.
Let Y be the sum of the #'s that come up on two fair dice. Find the pmf for Y.

Solution.
We note that Y can take values from 2 to 12. We compute the probabilities of these values using Axiom 1:

$$P(X=2) = \frac{|\{11\}|}{36} = \frac{1}{36},$$

$$P(X=3) = \frac{|\{12, 21\}|}{36} = \frac{2}{36},$$

$$P(X=4) = \frac{|\{13, 31, 22\}|}{36} = \frac{3}{36},$$

$$P(X=5) = \frac{|\{14, 41, 23, 32\}|}{36} = \frac{4}{36},$$

$$P(X=6) = \frac{|\{15, 51, 24, 42, 33\}|}{36} = \frac{5}{36},$$

$$P(X=7) = \frac{|\{16, 61, 25, 52, 34, 43\}|}{36} = \frac{6}{36},$$

$$P(X=8) = |\{26, 62, 35, 53, 44\}| = \frac{5}{36},$$

... (DIY: for $y = 9, 10, 11, 12$, we compute that $P(Y=y) = 4, 3, 2, 1$ respectively).

[Aside: a compact formula is

$$P(Y=y) = \frac{6-|7-y|}{36}. \text{]}$$

Check: the above probabilities add up to 1.

<u>Example 2.</u>

Flip 3 fair coins; let X = number of heads. Find the pmf for X.

<u>Solution.</u>

We have sample space

$$S = \{HHH, HHT, HTH, HTT, TTH, THT, HTT, TTT\}.$$

So

$P(X=0) = 1/8 = 12.5\%$, $P(X=1) = 3/8 = 37.5\%$,
$P(X=2) = 3/8 = 37.5\%$, $P(X=3) = 3/8 = 37.5\%$.