Lists, continued.

Recall: a list is an ordered sequence of items.

Guidelines/principles for counting lists:

1. Multiplication principle (MP):
   (a) If Thing 1 can be done in m ways and, for each of these ways, Thing 2 can be done in n ways, then Things 1 and 2 together can be done in mn ways.

   (b) The number of length-k lists that can be made from n items is
   (i) n^k allowing repetition;
   (ii) $n(n-1)(n-2)\cdots(n-k+1)$* disallowing repetition.

   *Sometimes denoted ${}_nP_k$ or $P(n,k)$.

   Note: a length-k list from n items, without repetition, is called a <u>k-permutation</u> of n.

2) (a) Addition principle (AP):
   If Thing 1 can be done in m ways and Thing 2 can be done in n ways, then the number of ways of doing Thing 1 <u>or</u> Thing 2 is $m+n$, <u>provided</u> you're not counting twice.

   (b) Inclusion-Exclusion Principle (IEP):
   If you <u>are</u> counting twice, <u>subtract</u>

to compensate.

3) Subtraction principle (SP):
number of lists with a property P
$=$ total number of lists
$-$ number without property P.

<u>Example</u>

How many 5-card <u>lists</u> can be dealt from a standard 52-card deck if:

(A) all have the same suit;
(B) Exactly one card is a 3;
(C) No card is a 3;
(D) At least one card is a 3;
(E) All have the same suit <u>or</u> no card is a 3 (or both).

<u>Solution.</u>

(A) The first card can be anything; its suit determines the suit of the other cards. So there are

$$52 \cdot 12 \cdot 11 \cdot 10 \cdot 9 = 617{,}760 \text{ such lists.}$$

(We've used MP(a).)

(B) $\exists\ 4 \cdot 48 \cdot 47 \cdot 46 \cdot 45$ lists where 3 is the <u>first</u> card, $48 \cdot 4 \cdot 47 \cdot 46 \cdot 45$ where it's the second, etc. Total:

$$5(4 \cdot 48 \cdot 47 \cdot 46 \cdot 45) = 93{,}398{,}400 \text{ such lists.}$$

(We've used MP(a) and AP(a).)

(c) 48 cards total are not 3's. So there are
$48 \cdot 47 \cdot 46 \cdot 45 \cdot 44 = 205{,}476{,}480$
lists with no 3's.
(We've used MP(a) or, equivalently, MP(b)(ii).)

(D)
$$\underbrace{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48}_{\text{all possible 5-card lists}} - \underbrace{48 \cdot 47 \cdot 46 \cdot 45 \cdot 44}_{\text{those with no 3's.}}$$
$= 106{,}398{,}720$ such lists, by MP(a) (or b(ii)) together with SP.

(E) The number of lists where all cards have the same suit <u>and</u> no card is a 3 is

$48 \cdot 11 \cdot 10 \cdot 9 \cdot 8 = 380{,}160$, by MP(a).

So, by the answers to parts (A) and (C) above, and by IEP, the number of lists with all cards the same suit <u>or</u> with no 3's is

$617{,}760 + 205{,}476{,}480 - 380{,}160$

$= 205{,}714{,}080.$