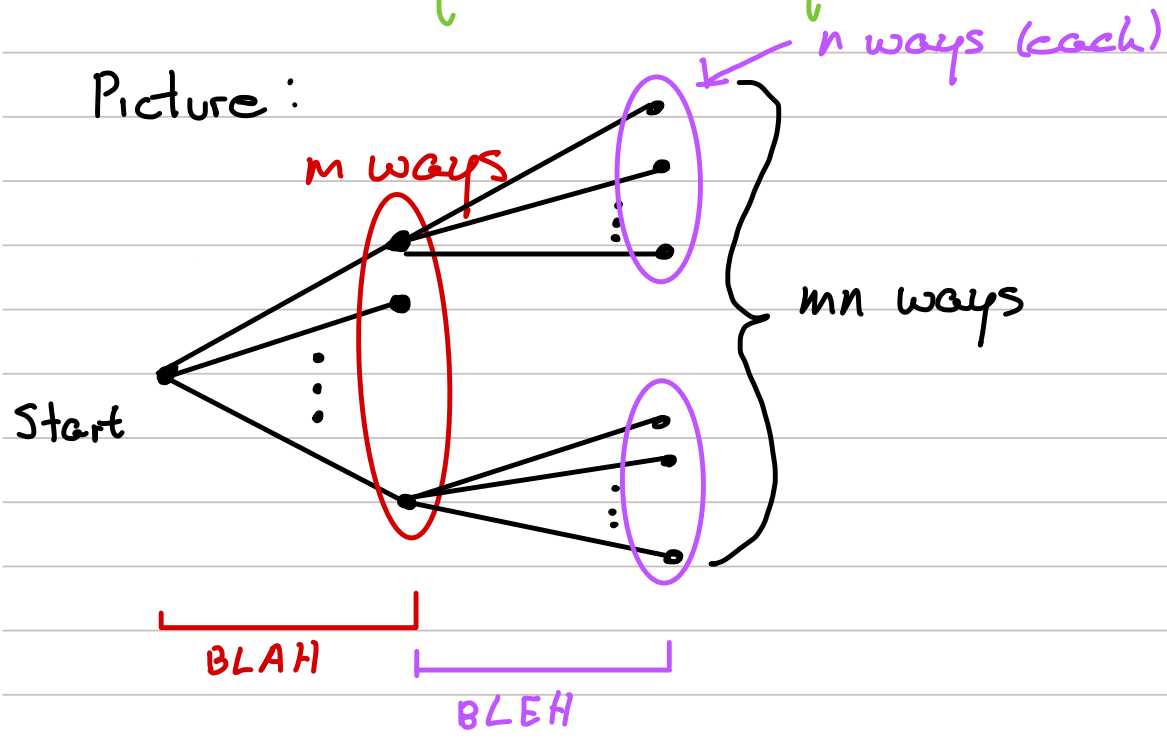
Combinatorics (a fancy word for counting).

The key is the Multiplication Principle (MP), which says:

> If there are m ways of doing BLAH and, for each of these ways, there are n ways of doing BLEH, then there are mn ways of doing BLAH followed by, or together with, BLEH.

Multiplication Principle (MP)

Picture:



We can use MP for two types of counting: (1) lists; (2) sets.

Today: (1) lists.
A list is an ordered sequence of items.

Example 1

How many six-symbol strings can be made from the letters A-Z and the digits

0-9, if:

(i) repetition is allowed;

(ii) repetition is not allowed;

(iii) repetition is not allowed, and the string must be 3 digits followed by 3 letters.

Solution

(i) $\exists\ 26+10=36$ choices for each symbol, yielding

$$36\cdot 36\cdot 36\cdot 36\cdot 36\cdot 36 = 36^6$$
$$= 2,176,782,336 \text{ codes.}$$

(ii) $36\cdot 35\cdot 34\cdot 33\cdot 32\cdot 31 = 1,402,410,240$ strings

(36 choices for first symbol, leaving only 35 choices for the second, etc.)

(iii) $10\cdot 9\cdot 8\cdot 26\cdot 25\cdot 24 = 11,232,000$ strings.

Note that, generally:

The number of k-element lists that can be made from a set with n elements is:

(i) n^k allowing repetition;

(ii) $n(n-1)(n-2)\cdots(n-k+1)$* disallowing repetition.

*Sometimes denoted ${}_nP_k$.

Some counting tips & tricks:

(a) If necessary, break things up into cases (then

add the counts from the separate cases).

(b) To count how many lists have a property P, it's sometimes easier to compute |{all lists}| - |{lists without property P}|.

(c) Be careful to subtract anything counted twice.

Example 2.

How many length-3 strings, w/o repetition, can be made from the letters A, B, C, D, E, if:

(a) the first or the second letter is an E;

(b) One letter is an E;

(c) The 2nd letter is a C or the 3rd is an E (or both).

Solution

(a)

$$\begin{array}{rl} & 1\cdot 4\cdot 3 \\ + & 4\cdot 1\cdot 3 \\ \hline & 24 \text{ strings} \end{array}$$

← first letter is an E

← second letter is an E

(b) $5\cdot 4\cdot 3 - 4\cdot 3\cdot 2 = 36$ strings

all strings — strings with no E

(or we can add cases: 12 strings have E as first letter; 12 have E as first; 12 have E as third).

(c) $4\cdot 1\cdot 3 + 4\cdot 3\cdot 1 - 3\cdot 1\cdot 1 = 21$ strings

2nd letter is a C

3rd letter is an E

we counted strings of the form *CE twice, so subtract once to compensate