Assignment 1

MATH 2001: Intro to Discrete Mathematics

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S-POP, Section B(i): Exercises B(i)-4 through B(i)-10.

Exercise B(i)-4. Supply a proof by contraposition of Proposition B(i)-1_{E}.

Proof. Proposition B(i)-1_{E} states "If n is an even number then $n-1$ is an odd number." Thus in order to prove it by contraposition, we will prove if $n-1$ is not an odd number, then n is not an even number. We can reprhase is to prove that if $n-1$ is an even number, then n is an odd number.

Let $n-1=2k$ to show that $n-1$ is equal to some even integer.

$$n-1=2k \tag{1}$$
$$n=2k+1 \tag{2}$$
$$\tag{3}$$

Let it be shown that if $n-1$ is an even number, then n is an odd number, or rather, "if $n-1$ is not an odd number, then n is not an even number". □

Exercise B(i)-5. Supply a direct proof of Proposition B(i)-2_{E}. Hint: Suppose m^2 is odd. Then we can write $m^2 = 2l+1$ where l is an integer. Now write $m = 2k+r$, where k is an integer, and r equals either 0 or 1. (Why can we write m this way?) Now we have two different ways of writing m^2; set them equal, do some algebra, and see what you can deduce about r.

Proof. Proposition B(i)-2_{E} states "If m^2 is an odd number, then m is an odd number." Let us represent this as $m^2 = 2l+1$ where $l \in \mathbb{Z}$. Now we will set m as an even and an odd number to check if aforementioned equation holds.

$$\text{Even case: } m = 2k \text{ where } k \in \mathbb{Z} \tag{1}$$
$$m^2 = 2l+1 \tag{2}$$
$$(2k)^2 = 2l+1 \tag{3}$$
$$4k^2 = 2l+1 \tag{4}$$
$$2(2k^2) = 2l+1 \tag{5}$$

Let it be shown that in the even case, the left-hand side of the equation must be an even number, while the right-hand side of the equation must be an odd number. Thus, thus case does not hold. We will now test the odd case.

$$\text{Odd case: } m = 2k+1 \text{ where } k \in \mathbb{Z} \tag{6}$$
$$m^2 = 2l+1 \tag{7}$$
$$(2k+1)^2 = 2l+1 \tag{8}$$
$$4k^2+4k+1 = 2l+1 \tag{9}$$
$$2(2k^2)+2(2k)+1 = 2l+1 \tag{10}$$
$$2(2k^2+2k)+1 = 2l+1 \tag{11}$$
$$2j+1 = 2l+1 \text{ where } j = 2k^2+2k \tag{12}$$
$$\tag{13}$$

Let it be shown that in the odd case, the left-hand side of the equation must be an odd number, while the right-hand side of the equation must also be an odd number. Or, that if m^2 is an odd number, then m must also be an odd number. □

Exercise B(i)-6. Using contraposition prove that, if n is not divisible by 4, then n is not divisible by 12.

Proof. To prove by contraposition, we will prove that if n is divisible by 12, then n is divisible by 4.

$$\frac{n}{12} = l \text{ where } n, l \in \mathbb{Z} \tag{1}$$
$$n = 12l \tag{2}$$
$$\frac{n}{4} = 3l \tag{3}$$

Let it be shown that assuming n and l are integers, there are integers n where $n/4$ is equal to 3 times some other integer l. □

Exercise B(i)-7. Consider the statement:

If x is odd, then x is divisible by 3.

Prove that this statement is false, using the method of counterexample.

Proof. Let $x = 7$. $3 \nmid 7$. Thus, the statement is false, since x being the odd number 7 cannot be divided by 3. □

Exercise B(i)-8. Consider the converse to the statement of Exercise B(i)-3(a). Is this converse statement true? If so, prove it. If not, show that its false by counterexample.

Proof. Exercise B(i)-3(a) states "Prove that, if $a \mid b$ and $a \mid c$, then $a \mid (b+c)$." The converse of that would be if $a \mid (b+c)$, then $a \mid b$ and $a \mid c$. Let's prove this false by counterexample. Let $a = 7$, $b = 40$, and $c = 9$.

$$a \mid (b+c) = 7 \mid (40+9) \tag{1}$$
$$7 \mid 49 \tag{2}$$
$$a \mid b = 7 \mid 40 \tag{3}$$
$$7 \nmid 40 \tag{4}$$
$$a \mid c = 7 \mid 9 \tag{5}$$
$$7 \nmid 9 \tag{6}$$
$$\tag{7}$$

Thus, let it be shown that the converse is not true, because while $a \mid (b+c)$ may be true, it does not imply $a \mid b$ and $a \mid c$. □

Exercise B(i)-9. Use the method outlined in Proposition B(i)-3 to show that an integer n is divisible by 6 if, and only if, n is both even and divisible by 3. Hint for one of the directions: note that, if n is divisible by 3, then n = 3k for some integer k. Now if n is also divisible by 2 that is, if n is even what does the equation n = 3k tell you about k? Use Exercise B(i)-1(b).

Proof. Let us first show that if n being both even and divisible by 3 implies that n is divisible by 6. We can show that n is both even and divisible by six by showing how it can be equal to some integer k multiplied by 2 and 3.

$$n = 2 * 3 * k \tag{1}$$
$$n = 6k \tag{2}$$
$$\frac{n}{6} = k \tag{3}$$

Since integer n is equal to some integer k times 6, it must mean that n divided by six is also an integer, thus showing that n is divisible by 6. Now let us show that if n is divisible by 6, then n is also even and divisible by 3.

$$\frac{n}{6} = k \text{ where } n, k \in \mathbb{Z} \tag{4}$$
$$n = 6k \tag{5}$$
$$n = 2 * 3 * k \tag{6}$$
$$\frac{n}{2} = 3k \tag{7}$$
$$\frac{n}{3} = 2k \tag{8}$$
$$\tag{9}$$

Let it be shown that for any integer n that is divisible by 6, that integer is also divisible by 2, yielding some integer k times 3, and also that n is divisible by 3, yielding some integer k times 2 (or in other words an even integer). □

Exercise B(i)-10. Let x be a real number. Use the method of proof shown in Proposition B(i)-3_{E} to show that $x^2 = 1$ iff $x = -1$ or $x = 1$. (Pretend you didnt know this already.) Hint: $x^2 = 1$ iff $x^2 - 1 = 0$. Now factor $x^2 - 1$, and use the fact that, for a, b real numbers, the product ab equals zero iff a = 0 or b = 0 (or both).

Proof.

$$x^2 = 1 \text{ iff} \tag{1}$$

$$x^2 - 1 = 0 \text{ iff} \tag{2}$$

$$(x+1)(x-1) = 0 \text{ iff} \tag{3}$$

$$x + 1 = 0 \iff x = -1 \text{ and } x - 1 = 0 \iff x = 1 \tag{4}$$

Thus $x^2 = 1$ iff $x = 1$ or $x = -1$ □