B(i)-4. If n is an even number then $n-1$ is odd. Contraposition:

Suppose $n-1$ is not an odd number. $n-1 = 2k$ for $n \in \mathbb{Z}$ and $k \in \mathbb{Z}$. Then $n = 2k+1$. n is now not an even number as $2k$ is even and $2k+1$ is odd. $\square$

$2l = 2m \quad 2l + 2m$

HW3 Discrete

~~B(i)-4. Suppose n is not an even number, so n = 2k+1, k ∈ ℤ. The n-1 = 2k+1-1 which equals 2k which as we know is an even number for k ∈ ℤ. □ So if n is even number n-1 is odd. □~~

B(i)-5. Proposition! If m^2 is an odd number, them m is an odd number.

Proof: Suppose m^2 is odd the we can write $m^2 = 2l+1$, wher $l \in \mathbb{Z}$. Suppose $m = 2k+r$, where $k \in \mathbb{Z}$ and r is equal to zero or one. Now can set these two values equal to each other

$$2l+1 = (2k+r)(2k+r)$$
$$2l+1 = 4k^2 + 2kr + 2kr + r^2$$
$$2l+1 = 4k^2 + 4kr + r^2$$
$$2l+1 = 2(2k^2+2kr) + r^2$$
$$2l+1 = 2x + r^2 \quad \text{where } x = 2k^2+2kr$$

as we see now r has to be 1 as an odd number can not equal an even number in the case of $2l+1 = 2x + r^2$. So if m^2 is an odd number, m is an odd number, as $2l+1 = 2x+1$ and both sides are odd. □

$5|(3+2)$ then $5|3$ $a|c$

B(i)-6. Suppose n is divisible by 12. Then $n = 12k$, for $k \in \mathbb{Z}$. This can be written as $4(3k)$ where $3k$ is an integer, so if n is divisible by 12 then it is also divisible by 4. $\square$

B(i)-7. If x is odd, then x is divisible by three. This statement is false as if we take 1 which is an odd number it is not divisible by 3 or a number like 7 which is also odd and not divisible by 3. $\square$

B(i)-8. Let a, b, c be integers. Recall that we say a divides b written $a|b$ if there exists an integer q such that $b = aq$. a.) proove that if $a|b$ and $a|c$, then $a|(b+c)$

Converse: If $a|(b+c)$ then $a|b$ and $a|c$. This statement is false suppose $a=5$, $b=3$, $c=2$ so $a|(b+c)$ is true as this equals $5|(3+2)$ which is $5|5$. However $a|b$ and $a|c$ are false as $a|b = 5|3$ and $a|c = 5|2$ which are both false statements! $\square$

$x^2-1=(x-1)(x+1)$ $\quad -1-1$ $\quad -2$

B(i) - d.d.) An integer n is divisible by 6, if and only if n is both even and divisible by 3. Suppose n is an integer divisible by 6, $n=6k$ for $k \in \mathbb{Z}$. n can be written as both $n=2(3k)$ showing it is even and $n=3(2k)$ showing it is divisible by 3.

b.) Suppose n is both even and divisible by 3. $n=6k$ for $k \in \mathbb{Z}$. n is even as $6k=2(3k)$. n is also divisible by 3 as $6k=3(2k)$. Both $2k$ and $3k$ are both integers in these examples. Since an integer n is divisible by 6, if and only if n is both even and divisible by 3 and n is both even and divisible by 3 if and only if n is divisible by 6. So n is divisible by 6 iff n is both even and divisible by 3. $\square$

8(i) - 10. Suppose x is a real number, then $x^2=1$ if and only if $x=-1$ or $x=1$. $(x^2=1)$ is $(x^2-1=0)$ is $((x-1)(x+1)=0)$ if we set $x=-1$ then $(-1-1)(-1+1)=0$ is equal to $(-2)(0)$ which is zero. If we set $x=1$ then $(x-1)(x+1)=(1-1)(1+1)=(0)(2)$ which is also zero. So $x^2-1=0$ if and only if $x=1$ or $x=-1$. $\square$

Dividend = Divisor × quotient ...

$x \in 3\mathbb{Z}+0 \cup 3\mathbb{Z}+1 \cup 3\mathbb{Z}+2$

$0 \leq r \leq b$

B(ii) - 5. Proposition: $\mathbb{Z} = (3\mathbb{Z}+0) \cup (3\mathbb{Z}+1) \cup 3\mathbb{Z}+2$,
Lets first show that $\mathbb{Z} \subseteq (3\mathbb{Z}+0) \cup (3\mathbb{Z}+1) \cup (3\mathbb{Z}+$
Let $x \in \mathbb{Z}$. By definition x is any integer.
x is any integer and $3\mathbb{Z}+0$, $3\mathbb{Z}+1$, and $3\mathbb{Z}+2$
are all integers as well. The division
algorithm explains that for any integer m
and positive integer b, there are unique
integers a and r with $m = ba + r$. Lets
say that $\mathbb{Z} = m$ in this case as m in ANY
integer. 3 is going to be the positive integer
b. So $\mathbb{Z} = 3a + r$. r must be equal to
or greater than zero but less than b.
There are no rules saying m can
not equal a in the division algorithm so we
may have $\mathbb{Z} = 3\mathbb{Z} + r$. r in this case must either
be 0, 1, or 2. This hits every case for any integer
$\mathbb{Z}$ when b in the division algorithm equals 3.
So if $x \in \mathbb{Z}$ then $x \in (3\mathbb{Z}+0) \cup (3\mathbb{Z}+1) \cup (3\mathbb{Z}+2)$.
Now lets show $(3\mathbb{Z}+0) \cup (3\mathbb{Z}+1) \cup (3\mathbb{Z}+2) \subseteq \mathbb{Z}$. Let
$x \in (3\mathbb{Z}) \cup (3\mathbb{Z}+1) \cup (3\mathbb{Z}+2)$. x is an integer which
means by the division algorithm $x = ba + r$, where b is
a positive integer, a is a unique integer and r
is $0 \leq r < b$. Lets set $b = 3$ so $x = 3a + r$. a can be
any unique integer so lets set $a = \mathbb{Z}$. $x = 3\mathbb{Z} + r$. r
must be $0 \leq r < b$ so r can be 0, 1, or 2. So
$x = 3\mathbb{Z}$ or $x = 3\mathbb{Z}+1$ or $3\mathbb{Z}+2$. So $x \in 3\mathbb{Z} \cup 3\mathbb{Z}+1 \cup 3\mathbb{Z}+2$
$3\mathbb{Z}$ or $3\mathbb{Z}+1$ or $3\mathbb{Z}+2$ are all integers so $x \in \mathbb{Z}$.
Since $\mathbb{Z} \subseteq 3\mathbb{Z} \cup 3\mathbb{Z}+1 \cup 3\mathbb{Z}+2$ and $3\mathbb{Z} \cup 3\mathbb{Z}+1 \cup 3\mathbb{Z}+2 \subseteq$
$\mathbb{Z} = 3\mathbb{Z} \cup 3\mathbb{Z}+1 \cup 3\mathbb{Z}+2$. $\square$

$3x = 3x+1$

14. 16.

$A \times (B \cup C) = (A \times B) \cup (A \times C)$

8(ii)-6. There is not as a and r must always be unique if b is a set integer by division algorithm.

8(ii)-7. Suppose $n \in X \cup (Y \cap Z)$. So by definition $n \in X$ or $n \in Y \cap Z$. If $n \in X$ then n also $n \in (X \cup Y) \cap (X \cup Z)$ by definition of union. If $n \in Y \cap Z$, $n \in Y$, $n \in Z$. So again by definition of union $n \in (X \cup Y) \cap (X \cup Z)$. So $X \cup (Y \cap Z) \subseteq (X \cup Y) \cap (X \cup Z)$.
Suppose $n \in (X \cup Y) \cap (X \cup Z)$. By definition $n \in X \cup Y$ and $n \in X \cup Z$. Also by definition $n \in X$ or $n \in Y$ and $n \in Z$. So n can be $n \in X \cap X$, $n \in X \cap Z$, $n \in Y \cap X$ or $n \in Y \cap Z$. So n is always going to be in either X or Y and Z so $n \in X \cup (Y \cap Z)$. So $(X \cup Y) \cap (X \cup Z) \subseteq X \cup (Y \cap Z)$. Therefore $X \cup (Y \cap Z) = (X \cup Y) \cap (X \cup Z)$. $\square$

14. Suppose $x \in (A \cup B) - C$. By definition $x \in A \cup B$ and $x \notin C$. Also by definition $x \in A$ or $x \in B$. So $x \in A - C$ or $x \in B - C$ so $x \in (A - C) \cup (B - C)$.
Suppose $x \in (A - C) \cup (B - C)$ so by definition $x \in A$ or $x \in B$ but $x \notin C$. So $x \in (A \cup B) - C$.
So $(A \cup B) - C = (A - C) \cup (B - C)$

16. Suppose $n \in A \times (B \cup C)$. By definition $n \in A \times B$ or $n \in A \times C$. So in this case $n \in (A \times B) \cup (A \times C)$ as we just explained n can be an element of either. Suppose $n \in (A \times B) \cup (A \times C)$. By definition $n \in A \times B$ or $n \in A \times C$, n is always an element of A and then either a cartesian product of B or C. So $n \in A \times (B \cup C)$. Therefore $A \times (B \cup C) \subseteq (A \times B) \cup (A \times C)$ and $(A \times B) \cup (A \times C) \subseteq A \times (B \cup C)$.

So $A \times (B \cup C) = (A \times B) \cup (A \times C)$