Homework 3

B(i) - 4

Assume $n-1$ is not an odd number. This means $n-1$ is even.
Then by definition $n-1 = 2k$ where $k \in \mathbb{Z}$
$n = 2k+1$
and the definition of odd numbers follows the previous form.
Therefor n is an odd number or rather not even.
so, by contraposition if $n-1$ is not odd then n is not even thus by logic equivalency
If n is even then $n-1$ is odd $\square$

B(i) - 5

~~Assume m^2 is odd then by definition~~
~~$m^2 = 2k+1$ $k \in \mathbb{Z}$~~
~~Lets expand and say $r=0$ or $r=1$~~
~~$m = 2k+r$ where $r \in [0,1]$~~
~~There are 2 cases~~
~~hence we can do the following~~
~~$2k$~~

Assume m^2 is odd then by definition of odd numbers
$m^2 = 2k+1$ where $k \in \mathbb{Z}$

Now lets write $m = 2\lambda + r$ where $\lambda \in \mathbb{Z}$
and r can be 0 or 1 (by division algorithm remainder is 0 or 1 if its even or odd)
Thus
$$2k+1 = (2\lambda + r)^2$$
$$2k+1 = 4\lambda^2 + 2\lambda r + r^2$$

~~case 1~~ if $r=0$

$$2k+1 = 4\lambda^2$$
$$2k+1 = 2(2\lambda^2)$$
$$2k+1 = 2l \quad \text{where } l \in \mathbb{Z}$$
Then by contradiction an odd number is not equal to an even number.
Thus $r = 1$ so
$$m^2 = 4\lambda^2 + 2\lambda + 1 = 2(2\lambda^2 + \lambda) + 1 \quad \text{where } P \in 2\lambda^2 + \lambda$$
hence $m^2 = 2P+1$ so ~~m~~ is odd.

Thus we can conclude that if m^2 is odd then m must also be odd based on the previous contradiction. And becase when $r = 1$ m follows $m = 2k+1$ which makes it odd □

B(i) - 6

Assume n is divisible by 12 then $n = 12k$.
Since $n = 12k$ and $12 = 4 \cdot 3$ we can write
$n = 4 \cdot 3k$
$n = 4\lambda \quad \lambda = 3k$

Since n is a integer $\cdot 4$ we know that n is divisible by 4. Then by contraposition and logical equivalence if n is not divisible by 4 then it is not divisable by 12. □

B(i) - 7

If x is odd, then x is divisable by 3. is false.
A simple counterexample is $x = 1$, for x to be divisible by 3 the modulus has to be equal to 0 since 1 mod 3 is = 1 we have proven that the pormentioned statement is false □

B(i) - 8

If $a|(b+c)$, then $a|b$ and $a|c$ is false.

let $a = 2$, $b = 3$ and $c = 3$

If 2 divides $(3+3) = 6$, then 2 should also divide b and c individually, since 2 dose not devide $b = 3$ or $c = 3$ we show that in either case the statement is false. □

B(i) - 9

1. ~~Assume~~

$\Rightarrow$) Assume n is divisible by 6

a) then $n = 6k$ $k \in \mathbb{Z}$

$n = 2 \cdot 3k$

$n = 2L$ $L = 3k$

thus n is even

b) $n = 6k$ and $6 = 3 \cdot 2$

$n = 3 \cdot 2k$ where $P = 2k$

$n = 3P$

since n is 3 times an integer it is divisible by 3

there for n is even and divisible by 3

n is both even and divisible by 3

2. $\Leftarrow$) Assume ~~n is not divisible by 6~~

then by the previous n must follow

$n = 2k$ where $k \in \mathbb{Z}$

if n is also divisible by 3 then it must follow

$n = 3\lambda$ where $\lambda \in \mathbb{Z}$

Since for this to hold

$2k = 3\lambda$ ~~integer must be some~~

~~k~~ k must be $= 3A$ and $\lambda = 2B$ where $A, B \in \mathbb{Z}$

in either case $n = 6A = 6B$ showing that n is divisible by 6

~~Since n is even and divisible by 3 and n is also divisible by 3~~

Since if n is divisible by 6, then ~~both~~ n is both even and divisible by 3 and if n is both even and divisible by 3 then n is divisible by 6 we have shown that n is divisible by 6 if, and only if, n is both even and divisible by 3. $\square$

B(i) = 10 Assume $x \in \mathbb{R}$ and that $x^2 = 1$
then $\sqrt{x^2} = \sqrt{1}$
$x = \pm 1$

1) $\Rightarrow$) Assume $x = 1$ then $x \cdot x = 1 \cdot 1$
so $x^2 = 1$

2) $\Leftarrow$) Assume $x = -1$ then $x \cdot x = (-1) \cdot (-1)$
so $x^2 = +1$

In either case $x^2 = 1$ so both $x = 1$ and $x = -1$ satisfy the solution $\square$

version II

B(i) - 10 Assume $x \in \mathbb{R}$ and that $x^2 = 1$
then $x^2 - 1 = 0$
$(x-1)(x+1) = 0$

case I $(x-1) = 0$
$x = 1$

case II $(x+1) = 0$
$x = -1$

So we have shown that $x = 1$ and $x = -1$ both satisfy the solution. $\square$

B(ii) - 5 Let $\mathbb{Z}$ be the set of integers and b_r denote the set of integers that follow $bg + r$ where $g \in \mathbb{Z}$.
The division algorithm follows this
~~Assume Let~~ formula $n = 3g + r$ where r can be $0 \le r \le 2$
for $\mathbb{Z} = 3\mathbb{Z} \cup 3\mathbb{Z}+1 \cup 3\mathbb{Z}+3$
~~Let x \in~~
By the division algorithm we can do the following
If $r = 0$, then $n = 3g$ which means $n \in 3\mathbb{Z}$ as it is a multiple of 3

If $r = 1$, then $n = 3g + 1$ which means that $n \in 3\mathbb{Z} + 1$

If $r=2$, then $n=3q+2$ which means $n \in 3\mathbb{Z}+2$

Since every integer n can be categorized in one of these 3 cases by ~~the~~ division algorithm we can say that

$\mathbb{Z} = 3\mathbb{Z} \cup 3\mathbb{Z}+1 \cup 3\mathbb{Z}+2$ □

B (ii) - 6

Going from the division algorithm we can say that an integer should fall in one of these ~~two~~ three cases

1. An integer in $3\mathbb{Z}$ cannot be in $3\mathbb{Z}+1$ or $3\mathbb{Z}+2$ because it leaves a remainder of 0 not 1 or 2

2. An integer in $3\mathbb{Z}+1$ cannot be in $3\mathbb{Z}$ or $3\mathbb{Z}+2$ because it specifically leaves a remainder of 1

3. An integer $3\mathbb{Z}+2$ cannot be in $3\mathbb{Z}$ or $3\mathbb{Z}+1$ because it specifically leaves a remainder of 2

The exclusivity show the disjoint property of the sets $3\mathbb{Z}$, $3\mathbb{Z}+1$ and $3\mathbb{Z}+1$ showing that there are no elements in common between all three cases □

B(ii) - 7

Let $X \cup (Y \cap Z) = (X \cup Y) \cap (X \cup Z)$

1) Assume $x \in X \cup (Y \cap Z)$
Then by definition of union $x \in X$ or $x \in (Y \cap Z)$.
~~We consider two cases~~ Then by defn of intersection $x \in Y$ and $x \in Z$

a) ~~X~~ We get two cases
* ~~X~~ If $x \in X$, then $x \in (X \cup Y)$ and $x \in (X \cup Z)$ by defn of ~~intersection~~ union.

b) $x \in Y \cap Z$ then $x \in Y$ and $x \in Z$. Since $x \in Y$ we have $x \in (X \cup Y)$ and by ~~defn operation~~ since $x \in Z$ we have and $x \in (X \cup Z)$ by defn of union and intersection

Therefore $X \cup (Y \cap Z) \subseteq (X \cap Y) \cup (X \cap Z)$

~~2)~~ Assume $x \in (X \cup Y) \cap (X \cup Z)$
Then by defn of union $x \in X \cup Y$ or $x \in X \cup Z$
and by defn of intersection $x \in Y$ and $x \in Z$

we get two cases

a) $x \in X$, Then since $x \in (Y \cap Z)$ we get
$x \in X \cup (Y \cap Z)$ by defin of union

b) $x \notin X$, Then since $x \in (Y \cap Z)$ we have
$x \in X \cup (Y \cap Z)$ by defn of union.

in either case we get $x \in X \cup (Y \cap Z)$

so $(X \cup Y) \cap (X \cup Z) \subseteq X \cup (Y \cap Z)$

Hence
$X \cup (Y \cap Z) = (X \cup Y) \cap (X \cup Z)$ $\square$

14)

1. Assume $x \in (A \cup B) - C$ this means $x \in A$ or $x B$ but $x \in C$ by dfn of union

a) $x \in A$ and $x \notin C$ then $x \in A - C$. Thus $x \in (A-C) \cup x \in (B-C)$ by defn of union

b) $x \in B$ and $x \notin C$ then $x \in B - C$. Thus $x \in (A-C) \cup x \in (B-C)$ by defn of union

In either case $x \in (A-C) \cup (B-C)$
so
$(A \cup B) - C \subseteq (A-C) \cup (B-C)$

2. Assume $x \in (A-C) \cup (B-C)$
Then by defn $x \in A-C$ or $x \in B-C$

a) if $x \in A-C$ then $x \in A$ but $x \notin C$. since $x \in A$ then $x \in (A \cup B)$ by defn of union
so $x \in (A \cup B) - C$

b) if $x \in B-C$, then $x \in B$ but $x \notin C$. since $x \in B$ then $x \in A \cup B$ by defn of union.
so $x \in (A \cup B) - C$

In either case $x \in (A \cup B) - C)$

so $(A-C) \cup (A-B) \subseteq (A \cup B) - C$

Hence ~~$(A \cup B) - C = (A \cup B) \cup (A \cup$~~

$(A \cup B) - C = (A-C) \cup (B-C)$ $\square$

1(b)

~~Assume $(x,y) \in A \cdot (B \cup C)$ then this means $x \in A$ and $x \in (B \cup C)$. Then by the defn of union $x \in B$ or $x \in C$.~~

~~or Tp $x \in$~~

Assume $(x,y) \in A \times (B \cup C)$ then this means that $x \in A$ and $y \in (B \cup C)$. Then by defn of union $y \in B$ or $y \in C$

1\.

a) $y \in B$, Then $x \in A$ and $(x,y) \in (A \times B) \cup (A \times C)$ by defn of union

b) $y \in C$. Then $x \in A$ and $(x,y) \in (A \times B) \cup (A \times C)$ by defn of union

In either case $(x,y) \in (A \times B) \cup (A \times C)$ so

2\. ~~Assume~~ $A \times (B \cup C) \subseteq (A \times B) \cup (A \times C)$

2\. Assume $(x,y) \in (A \times B) \cup (A \times C)$ Then by defn of union $(x,y) \in A \times B$ or $(x,y) \in A \times C$

a) $(x,y) \in A \times B$ so $x \in A$ and $y \in B$ so $y \in (B \cup C)$ by defn of union. Thus $(x,y) \in A \times (B \cup C)$

b) $(x,y) \in A \times C$ so $x \in A$ and $y \in C$ so $y \in (B \cup C)$ by defn of union. Thus $(x,y) \in A \times (B \cup C)$

In either case we have $(x,y) \in A \times (B \cup C)$ so

$$(A \times B) \cup (A \times C) \subseteq A \times (B \cup C)$$

Hence $A \times (B \cup C) = (A \times B) \cup (A \times C)$ $\square$