Homework 1

Fazzy Wright
Section 004

TBOP

A(1, 3, 6, 9, 14) B(17, 18, 26, 27) C(29, 32, 34, 38)

1) $\{5x - 1) : x \in \mathbb{Z}\}$

$x=0: 0-1 = -1$

$x=1: 5-1 = 4$

$x=-1: -5-1 = -6$

$x=2: 10-1 = 9$

$x=-2: -10-1 = -11$

$x=3: 15-1 = 14$

$x=-3: -15-1 = -16$

$\{..-16, -11, -6, -1, 4, 9, 14, 19\}$

3) $\{x \in \mathbb{Z} : -2 \leq x < 7\}$

$\{-2, -1, 0, 1, 2, 3, 4, 5, 6\}$

6) $\{x \in \mathbb{R} : x^2 = 9\}$

$\{-3, -\sqrt{9}, 3, \sqrt{9}\}$

9) $\{x \in \mathbb{R} : \sin \pi x = 0\}$

$\{..., -2, -1, 0, 1, 2, 3, 4\}$

14) $\{5x : x \in \mathbb{Z}, |2x| \leq 8\}$

$|2x| \leq 8$

$2 \cdot 1 = 2$ $2 \cdot 0 = 0$ $2 \cdot -1 = -2$

$2 \cdot 2 = 4$ $2 \cdot 3 = 6$ $2 \cdot 4$

$\{..-4, -2, 0, 2, 4, 6, 8\}$

B) 17) $\{2, 4, 8, 16, 32, 64..\}$

$2^1 = 2$ $2^2 = 4$ $2^3 = 2 \cdot 2 \cdot 2 = 8$ $2^4 = 16 \cdot 2 = 32 \cdot 2 = 64$

$\{2^x : x \in \mathbb{N}\}$

18) $\{0, 4, 16, 36, 64, 100...\}$

$4 \cdot 0 = 0$ $4 + 12 = 16$ $12 \cdot 1 = 12$

$4 \cdot 4 = 16$ $4 + 32 = 36$ $12 \cdot 3 = 36$

$4 \cdot 9 = 36$ $4 + 60 = 64$ $12 \cdot 5 = 60$

$4 \cdot 16 = 6$ $4 + 96 = 100$ $12 \cdot 8 =$

$4 \cdot 25 = 100$ $2^0 = 1$ $2^2 = 4$ $2^4 = 16$ 2^6

26) $\{... \frac{1}{27}, \frac{1}{9}, \frac{1}{3}, 1, 3, 9, 27...\}$

$3^0 = 1$ $3^{-3} = \frac{1}{27}$

$3^1 = 3$ $3^{-2} = \frac{1}{9}$

$3^2 = 9$ $3^{-1} = \frac{1}{3}$

$3^3 = 27$

$\{3^x : x \in \mathbb{Z}\}$

27) $\{..-\pi, -\frac{\pi}{2}, 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi, \frac{5\pi}{2}...\}$

$-\pi \to \frac{-\pi}{2} : \frac{\pi}{2}$

$\{\frac{k\pi}{2} : k \in \mathbb{Z}\}$

C) 29) $|\{\{1\}, \{2, \{3, 4\}\}, \emptyset\}|$

$x = 3$

32) $|\{\{\{1, 4\}, a, b, \{\{3, 4\}\}, \{\emptyset\}\}|$

$x = 1$

34) $|\{x \in \mathbb{N} : |x| < 10\}|$

$x = -10$

38) $|\{x \in \mathbb{N} : 5x \leq 20\}|$

$x = 4$

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5) $x, y \in \mathbb{Z}$. If x & y are odd, then xy is odd

Proof

Assume x is even, $x, y \in \mathbb{Z}$

Then $x = 2a$ for some int. a

Then $xy = (2a)(y) = 2(ay)$

$\boxed{\text{Therefore } xy = 2b \text{ where } b \text{ is int } ay \therefore xy \text{ is even}}$

7) Theory: Suppose $a, b \in \mathbb{Z}$. If $a|b$ then $a^2|b^2$

Assume $a|b$

By divisibility $b = ac$ for some int c

$b^2 = a^2c^2$

Then $b^2 = a^2d$, $d = c^2 \in \mathbb{Z}$

$\boxed{\text{By divisibility } a^2|b^2}$

14) If $n \in \mathbb{Z}$ then $5n^2 + 3n + 7$ is odd

Proof

Assume n is even

Then $n = 2k$ for some $k \in \mathbb{Z}$

$5n^2 + 3n + 7 = 5(2k)^2 + 3(2k) + 7$

$= 20k^2 + 6k + 6 + 1$

$= 2(10k^2 + 3k + 3) + 1$

$= 2\ell + 1$ $\ell = 10k^2 + 3k + 3 \in \mathbb{Z}$

$\boxed{\text{Therefore } 5n^2 + 3n + 7 \text{ is odd}}$

SPOP

B(i)-1) a) Theorem: Prove that the sum of two odd numbers is even

Proof:

- Assume m is odd
- Then $m = 2k + 1$ for k is some int.
- $m + m = (2k+1) + (2k+1)$

Bi-2) a) Theory: If n is even, then n^2 is divisible by 4

Proof:

- Assume n is even
- By definition $n = 2k$ for some int. k
- so $n^2 = 4k^2$ for some int $k \in \mathbb{Z}$
- $\boxed{\text{So } n^2 \text{ is divisible by definition of multiplication \& divisibility}}$

b) Theory: If n is odd number, then $n^2 - 1$ is divisble by 4

- Assume n is odd
- then by definition $n - 1 = 2k - 1$ for some int k
- Then $n^2 - 1 = 4k^2 - 1$ for $k \in \mathbb{Z}$
- $\boxed{\text{Then by definition of division } n^2 - 1 \text{ is divisible by } 4}$

Bi-3) a) Theory: If $a|b$ & $a|c$ then $a|(b+c)$

Proof: Assume $a|b$ & $a|c$

- then there exists q such that $b = aq$ & ℓ such that $c = a\ell$
- Then $b = aq$ & $c = a\ell$

  $a = \frac{b}{q}$ $a = \frac{c}{\ell}$
- Then $a = \frac{b}{q} + \frac{c}{\ell}$
- $\boxed{a|(b+c)}$

b) Theory: $a|b$ then $a|nb$ for any int n

Assume: $a|b$

- Then