(T-BoP chapter 4: Question 14 is continued at the bottom of HW1 pg. 2)

HW1

TBoP Section 1.1:

A.) (1) $\{5x-1 : x \in \mathbb{Z}\}$
$= \{\ldots, -11, -6, -1, 4, 9, 14, \ldots\}$

(3) $\{x \in \mathbb{Z} : -2 \leq x < 7\}$
$= \{-2, -1, 0, 1, 2, 3, 4, 5, 6\}$

(6) $\{x \in \mathbb{R} : x^2 = 9\}$
$= \{-3, 3\}$

(9) $\{x \in \mathbb{R} : \sin \pi x = 0\}$
$= \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

(14) $\{5x : x \in \mathbb{Z}, |2x| \leq 8\}$ $|2x| \leq 8 \Rightarrow -4 \leq x \leq 4$
$= -4, -3, -2, -1, 0, 1, 2, 3, 4$
$\boxed{= \{-20, -15, -10, -5, 0, 5, 10, 15, 20\}}$

B.) (17) $\{2^n : n \in \mathbb{N}\}$

(18) $\{n \in \mathbb{R} : x \in \mathbb{Z}, 0 \leq x, n = 4x^2\}$

(26) $\{3^n : n \in \mathbb{Z}\}$

(27) $\{\frac{n\pi}{2} : n \in \mathbb{Z}\}$

C.) (29) 3

(32) 5

(34) 9

(38) 4

TBoP, chapter 4 (126-127)

(5) Proposition: If x is even, then xy is even.

Proof:
Suppose $x \in \mathbb{Z}, y \in \mathbb{Z}$ and x is even.
By definition, $x = 2n$ for some $n \in \mathbb{Z}$.
So $xy = 2(ny) = 2l$,
where $l = ny$.
Therefore, xy is even. □

(7) Proposition: If $a|b$, then $a^2|b^2$.

Proof:
Suppose $a \in \mathbb{Z}, b \in \mathbb{Z}$.
Then by definition $a|b$ means $b = an$ for some $n \in \mathbb{Z}$.
So $b^2 = a^2 n^2$
Then $b^2 = a^2 k$, where $k = n^2 \in \mathbb{Z}$.
Therefore, $a^2 | b^2$. □

(14) Proposition: If $n \in \mathbb{Z}$, then n^2+3n+4 is even.

Proof:
Suppose $n \in \mathbb{Z}$.
We consider two cases:
a) n is even. Then $n = 2k$ for some $k \in \mathbb{Z}$.
Then, $n^2+3n+4 = (2k)^2 + 3(2k) + 4 = 4k^2 + 6k + 4 = 2(2k^2+3k+2)$
$= 2l$, where $l = 2k^2+3k+2$.
So n^2+3n+4 is even.
b) n is odd. Then by definition $n = 2k+1$ for some $k \in \mathbb{Z}$.
So $n^2+3n+4 = (2k+1)^2 + 3(2k+1) + 4 = 4k^2+4k+1+6k+3+4 = 4k^2+10k+8$
$= 2(2k^2+5k+4) = 2l$, where $l = 2k^2+5k+4$. $\longrightarrow$

HW1 Pg. 2

S-POP, Part B(i):

B(i) - 1:

a.) Theorem:

If n is an odd number and m is an odd number then $n+m$ will be an even number.

Proof:

Assume n is an odd number and assume m is an odd number.
Then by definition, $n = 2k+1$ for some $k \in \mathbb{Z}$.
Then by definition, $m = 2j+1$ for some $j \in \mathbb{Z}$.
So $n+m = 2k+1 + 2j+1 = 2(k+j+1) = 2\ell$
where $\ell = k+j+1$
so the sum of two odd numbers n and m is even. $\square$

b.) Theorem:

If n is an odd number and m is an odd number then $n \cdot m$ will result in an odd number.

Proof:

Assume n is an odd number and assume m is an odd number.
Then by definition, $n = 2k+1$ for some $k \in \mathbb{Z}$.
Then by definition, $m = 2j+1$ for some $j \in \mathbb{Z}$.
So $n \cdot m = (2k+1)(2j+1) = 4kj + 2k + 2j + 1 = 2(2kj + k + j) + 1$
$= 2\ell + 1$
where $\ell = 2kj + k + j$
So the product of two odd numbers n and m is odd. $\square$

2.4 continued (from Page 1)

So n^2+3n+4 is even.
So n must be even or odd, we see that n^2+3n+4 is always even. $\square$

HWK 1 Pg. 3

B(i)-2:

a.) Theorem:

If n is an even number, then n^2 is divisible by 4.

Proof:

Assume n is an even number.
Then by definition, $n = 2k$ for some $k \in \mathbb{Z}$.
So $n^2 = (2k)^2 = 4k^2$.
Thus, $n^2 = 4k^2$ which is divisible by 4.
So n^2 is divisible by 4 if n is even. $\square$

b.) Theorem:

If n is an odd number, then n^2-1 is divisible by 4.

Proof:

Assume n is an odd number.
Then by definition, $n = 2k+1$ for some $k \in \mathbb{Z}$.
So $n^2-1 = (2k+1)^2-1 = 4k^2+4k = 4(k^2+k) = 4l$
where $l = k^2+k$
So if n is odd the n^2-1 is divisible by 4. $\square$

B(i)-3:

a.) Theorem:

If $a|b$ and $a|c$, then $a|(b+c)$.

Proof:

Suppose $a \in \mathbb{Z}$, $b \in \mathbb{Z}$, $c \in \mathbb{Z}$ and that $a|b$ and $a|c$.
Then by definition, $b = an$ for some $n \in \mathbb{Z}$.
Then by definition, $c = ak$ for some $k \in \mathbb{Z}$.
So, $b+c = an + ak = a(n+k)$.
Therefore, $a|(b+c)$. $\square$

Theorem:

If $a|b$ then $a|nb$ for any integer n.

Proof:

Suppose $a \in \mathbb{Z}$, $b \in \mathbb{Z}$ and $a|b$
Then by definition, $b = ak$ for some $k \in \mathbb{Z}$.
Then by definition, $nb = am$ for some $m \in \mathbb{Z}$.
Since $b = ak$ then $nb = nak$.
So $m = nk$.
Thus, $nb = am$.
Therefore, $a|nb$. $\square$