Assignment 1

MATH 2001: Intro to Discrete Mathematics
January 24, 2024 Nicholas Nguyen

S-POP, Part B(i): Exercises B(i)-1, 2, 3.

Exercise B(i)-1.

(a) Prove that the sum of two odd numbers is even.

Proof. Let the sum of two odd numbers be represented by $(2a+1)+(2b+1)$ where $a, b \in \mathbb{R}$.

$$(2a+1)+(2b+1) \quad (1)$$
$$2b+2a+2 \quad (2)$$
$$2(a+b+1) \quad (3)$$
$$2k \text{ where } k=(a+b+1) \quad (4)$$

So by the definition of an even number, the sum of two odd numbers is even. □

(b) Prove that the product of two odd numbers is odd.

Proof. Let the product of two odd numbers be represented by $(2a+1)*(2b+1)$ where $a, b \in \mathbb{R}$.

$$(2a+1)*(2b+1) \quad (1)$$
$$4ab+2a+2b+1 \quad (2)$$
$$2(2ab+a+b)+1 \quad (3)$$
$$2k+1 \text{ where } k=2ab+a+b \quad (4)$$

So by the definition of an odd number, the product of two odd numbers is odd. □

Exercise B(i)-2.

(a) Prove that, if n is an even number, then n^2 is divisible by 4.

Proof. Assume n is an even number where $n \in \mathbb{R}$. Let $n=2k$ where $k \in \mathbb{R}$.

$$n=2k \implies n^2=4k^2 \quad (1)$$
$$\frac{n^2}{4}=k^2 \implies 4|n^2 \quad (2)$$

So n^2 is shown to be divisible by 4 assuming n is an even number and $n, k \in \mathbb{R}$. □

(b) Prove that, if n is an odd number, then n^2-1 is divisible by 4.

Proof. Assume n is an odd number where $n \in \mathbb{R}$. Let $n = 2k - 1$ where $k \in \mathbb{R}$.

$$n = 2k - 1 \implies n^2 = 4k^2 - 4k + 1 \tag{1}$$
$$n^2 - 1 = 4k^2 - 4k \tag{2}$$
$$n^2 - 1 = 4(k^2 - k) \tag{3}$$
$$\frac{n^2 - 1}{4} = k^2 - k \implies 4|n^2 - 1 \tag{4}$$

So $n^2 - 1$ is shown to be divisible by 4 assuming n is an odd number and $n \in \mathbb{R}$. $\square$

Exercise B(i)-3. Let a, b, and c be integers. Recall that we say a divides b, written $a|b$, if there exists an integer q such that $b = aq$.

(a) Prove that, if $a|b$ and $a|c$, then $a|(b + c)$.

(b) Prove that, if $a|b$, then $a|nb$ for any integer n.

T-BOP, Section 1.1 (pages 7–8): Exercises A(1, 3, 6, 9, 14), B(17,18, 26, 27), C(29, 32, 34, 38)

Exercises A. Write each of the following sets by listing their elements between braces.

(1) $\{5x - 1 : x \in \mathbb{Z}\} = \{..., -11, -6, -1, 4, 9, 14, ...\}$

(3) $\{x \in \mathbb{Z} \ : -2 \leq x < 7\} = \{-2, -1, 0, 1, 2, 3, 4, 5, 6\}$

(6) $\{x \in \mathbb{R} \ : x^2 = 9\} = \{-3, 3\}$

(9) $\{x \in \mathbb{R} \ : \sin \pi x = 0\} = \{..., -3, -2, -1, 0, 1, 2, 3, ...\}$

(14) $\{5x : x \in \mathbb{Z}, |2x| \leq 8\} = \{-20, -15, -10, -5, 0, 5, 10, 15, 20\}$

Exercises B. Write each of the following sets in set-builder notation.

(17) $\{2, 4, 8, 16, 32, 64...\} = \{2^x : x \in \mathbb{N}\}$

(18) $\{0, 4, 16, 36, 64, 100, ...\} = \{X_i : X_1 = 0, X_{i+1} = 8(i - 2) + X_i + 4, i \in \mathbb{N}\}$

(26) $\{2, 4, 8, 16, 32, 64...\} = \{3^x : x \in \mathbb{Z}\}$

(27) $\{..., -\pi, -\frac{\pi}{2}, 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi, \frac{5\pi}{2}, ...\} = \{\frac{x\pi}{2} : x \in \mathbb{Z}\}$

Exercises C. Find the following cardinalities.

(29) $|\{\{1\}, \{2, \{3, 4\}\}, \emptyset\}| = 3$

(32) $|\{\{\{1, 4\}, a, b, \{\{3, 4\}\}, \{\emptyset\}\}\}| = 1$

(34) $|\{x \in \mathbb{N} : |x| < 10\}| = 9$

(38) $|\{x \in \mathbb{N} : 5x \leq 20\}| = 4$

T-BOP, Chapter 4 (pages 126–127): 5, 7, 14.

Exercises

(5) Suppose $x, y \in \mathbb{Z}$. If x is even, then xy is even.

Proof. Assume $x, y \in \mathbb{Z}$ and x is even. We will xy as $2k * y$.

$$2k * y \tag{1}$$
$$2(ky) \tag{2}$$

Thus, but the definition of an even number, given that $x, y \in \mathbb{Z}$ and x is even, xy is even. □

(7) Suppose $a, b \in \mathbb{Z}$. If $a|b$, then $a^2|b^2$.

Proof. Assume $a, b \in \mathbb{Z}$.

$$\frac{b}{a} = n \tag{1}$$
$$b = n * a \tag{2}$$

□