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Math 2001

SPOP B(i)

B(i)-1 Prove sum of 2 odd ints = even

a) let x, y be odd integers represented as
$x = 2n+1$
$y = 2m+1$

$$x+y = 2n+1+2m+1$$
$$= 2n+2m+2$$
$$= 2(n+m+1) \qquad k = (n+m+1)$$
$$= 2k$$

Thus the sum of 2 odd integers is even $\square$

b) let x, y be odd integers represented as
$x = 2n+1$
$y = 2m+1$

$$x \cdot y = (2n+1)(2m+1)$$
$$= 4nm + 2n + 2m + 1 \qquad k = 2nm+n+m$$
$$= 2(2nm+n+m)+1$$
$$= 2k+1$$

So the product of 2 odd integers is odd $\square$

B(i)-2 a) if n is even n^2 is divisible by 4

let $n \in \mathbb{Z}$ and be even

n can represented as $n = 2k$ $\quad k \in \mathbb{Z}$

$$(2k)^2 = 2k \cdot 2k = 4k^2$$

and since $4k^2$ is divisible by 4 then if n is even n^2 is divisible by 4

b) if n is odd n^2-1 is divisible by 4

let $n \in \mathbb{Z}$ and odd so it's represented as $n=(2k+1)$

n^2-1 $=(2k+1)^2-1$

$=(2k+1 \cdot 2k+1)-1$

$=(4k^2+4k+1)-1$

$=4k^2+4k$

$=4(k^2+k)$ $m=k^2+k$

n^2-1 $=4m$

Since $4m$ is divisible by 4 n^2-1 is divisible by 4

B(i)-3 $a,b,c \in \mathbb{Z}$ $a|b$ $b=aq$

a) let $a,b,c \in \mathbb{Z}$ and $a|b$ and $a|c$

$a|b \Rightarrow b=aq$ $a|c \Rightarrow c=ax$ $q,x \in \mathbb{Z}$

$b+c = aq+ax$

$b+c = a(q+x)$ $q+x=w$

$b+c=aw$ Thus proving if $a|b$ & $a|c$ $a|(b+c)$ $\square$

b) let $a,b \in \mathbb{Z}$ and $a|b$

$b=aq$ $nb=n(aq)$ $n \in \mathbb{Z}$

$nb = a(nq)$ $w=nq$

$nb = aw$

this proves that if $a|b$ then $a|nb$ for any integer n

TBOP 5) let $x, y \in \mathbb{Z}$ and x be even

since x is even it can be represented by $x = 2K$

There are 2 cases, 1 where y is even and 1 where y is odd

y is even case: since y is even it can be represented by $y = 2n$

$$xy = (2K)(2n)$$

$$= 4Kn \qquad m = 2Kn$$

$$= 2m$$

This shows when y & x are even the product is even

y is odd

$y = 2n+1$

$$xy = (2K)(2n+1)$$

$$= 4Kn + 2K$$

This proves when x is even and y is odd xy is even

$\square$

7) let $a, b \in \mathbb{Z}$ and $a \mid b$

Since $a \mid b$ $\quad b = aq \quad q \in \mathbb{Z}$

$$b^2 = (aq)^2$$

$$b^2 = a^2 q^2 \qquad k = q^2$$

$$b^2 = a^2 k \qquad k \in \mathbb{Z}$$

This proves if $a \mid b$ Then $a^2 \mid b^2$ $\square$

Proves: $n \in \mathbb{Z}$ $\quad 5n^2 + 3n + 7$ is odd

14) let $n \in \mathbb{Z}$

n is even

$n = 2k$ sub in $2k$

$5(2k)^2 + 3(2k) + 7$

$5(4k^2) + 6k + 7$

$20k^2 + 6k + 7$

$2(10k^2 + 3k) + 7$ $\quad m = 10k^2 + 3k$

$2m + 7$

Proves when n is even $5n^2 + 3n + 7$ is odd

n is odd

$n = 2k+1$ sub in $2k+1$

$5(2k+1)^2 + 3(2k+1) + 7$

$5(4k^2 + 4k + 1) + 6k + 3 + 7$

$20k^2 + 20k + 5 + 6k + 3 + 7$

$= 20k^2 + 26k + 15$

$= 2(10k^2 + 13k) + 15$ $\quad m = 10k^2 + 13k$

$= 2m + 15$

Proves when n is odd $5n^2 + 3n + 7$ is odd

$\square$

TBOP A) 1. $\{\ldots,-16,-11,-6,-1,4,9,14,19,\ldots\}$

3 $\{-2,-1,0,1,2,3,4,5,6,7\}$

6 $\{-3,3\}$

9 $\{\ldots,-1,0,1,\ldots\}$ $\sin\pi = 0$

14 $\{$?

B) 17 $\{2^n : n \in \mathbb{N}\}$

18 $\{$ $\}$

20

27

C) 29. 3

32. 1

34. 10

38. 5