$(n+1)(n+1)$ $n^2+n+n+1$
n^2+2n

HW1 Discrete

S-POP

3(i)-1 a. Assume n is an even number, we can write $n=2k$ for some integer k. But then $n-1=2k-1$ so $2k-1$ is odd. If we add two odd numbers such that $(2k-1)+(2k-1) = 2k-1+2k+1 = 4k$. 4 is an even number times any integer k which implies $4k$ is even. $\square$

b. Assume n is an even number, we can write $n=2k$ for some integer k. But then $n+1 = 2k+1$, so $n+1$ is odd. If we do a product of two odd numbers that implies $(n+1)(n+1) = n^2+n+n+1 = n^2+2n+1$. $n^2 = (2k)^2 = (2k)(2k) = 4k^2$. As we discussed prior $4k$ is an even number and $4k^2$ is also even as k^2 is just another integer. $2n = 2(2k) = 4k$. 2 is a common factor of all even numbers, so if we add we add two even numbers $2x+2y$ for some integers x and $y \Longrightarrow 2x+2y = 2(x+y)$. $x+y$ is another integer so adding two even numbers equals an even number $\Longrightarrow n^2+2n$ is an even number which we will write as m. $m+1$ is an odd number meaning the product of two odd numbers is even.

T-80P read 3–7 and 112–125

8(i)-2 a. If n is an even number than $n=2k$ for any integer k. $n^2=(2k)^2=(2k)(2k)=4k^2$. We can see that k^2 is just another integer times 4 which implies $4k^2$ has a factor of 4 and $4k^2=n^2$. □

b. If n is an odd number than $n=2k+1$. $n^2=(2k+1)(2k+1)=4k^2+2k+2k+1=4k^2+4k+1$. so $n^2-1=(4k^2+4k+1)-1=4k^2+4k$. $4k^2$ and $4k$ are implied to both be factors of 4 and adding two factors of 4 is also gonna be a factor of 4. so n^2-1 is divisible by 4. □

8(i)-3. a. If $a|b \Rightarrow \frac{b}{a}=q$, where q is an integer. $a|b \Rightarrow \frac{b}{a} \Rightarrow b=aq$. If $a|b$ and $a|c$ that means a is a factor of both b and c. If you add together two integers that have a factor of a then the result will also be a factor of a. This implies that $(b+c)$ includes a as a factor $\Rightarrow a|(b+c)$. □

b. If $a|b$ that implies that a is a factor of b. That means no matter how many times you add up b, a will will always be a factor, so $a|bn$ for any integer n. □

$\frac{1}{x} = 4$

A (1, 3, 6, 9, 14)
B (17, 18, 26, 27)
C (29, 32, 34, 38

	0	4	16	36	64	100
4	0	1	4	9	16	25
2	0	2	8	18	32	50
J	0	2	4	6	8	10

T-307 section 1.1

A. 1. $\{5x-1 : x \in \mathbb{Z}\} = \{-1, 4, 9, 14, \ldots\}$

3. $\{x \in \mathbb{Z} : -2 \leq x < 7\} = \{-2, -1, 0, 1, 2, 3, 4, 5, 6\}$

6. $\{x \in \mathbb{R} : x^2 = 9\} = \{3\}$

9. $\{x \in \mathbb{R} : \sin(\pi x) = 0\} = \{0\}$

14. $\{5x : x \in \mathbb{Z}, |2x| \leq 8\} = \{0, 1, 2, 3, 4, -1, -2, -3, -4\}$

B. 17. $\{2^x : x \in \mathbb{N}\}$

18. $\{x^2 : x \in \mathbb{Z}, 2x\}$

26. $\{3^x : x \in \mathbb{Z}\}$

27. $\{\sin(x\pi/2) : x \in \mathbb{Z})$

C. 29. 1

32. 1

34. 9

38. 4

Chapter 4 Read 112-125) 5, 7, 14

5. Suppose x is even, then $x = 2k$ for $k \in \mathbb{Z}$.
suppose y is odd, then $y = 2l+1$ for $l \in \mathbb{Z}$.

$xy = (2k)(2l+1)$
$xy = 4kl + 2k$
$xy = 2(2kl + k)$
$xy = 2m$ where $m = 2kl + k$, m is just another integer so xy is even.

Suppose x is even, then $x = 2k$ for $k \in \mathbb{Z}$.
Suppose y is even, then $y = 2l$ for $l \in \mathbb{Z}$.

$xy = (2k)(2l)$
$xy = 4kl$
$xy = 2(2kl)$
$xy = 2m$ where $m = 2kl$, m is just another integer so xy is even. ∎

$(2k)(2k)$

$(2k+1)(2k+1)$ $\quad 4k^2$

$4k^2+2k+2k+1$ $\quad 4k^2+4k+1$

7. $a|b \Rightarrow b = ax$ for some integer x.
if we square each side then

$b^2 = (ax)^2$

$b^2 = a^2x^2$

$\frac{b^2}{a^2} = x^2$ for some integer x.

An integer times itself is still an integer, so $a|b \Rightarrow a^2|b^2$. ∎

14. If $n \in \mathbb{Z}$, then $5n^2+3n+7$ is odd.
Suppose n is even, so $n = 2k$ for $k \in \mathbb{Z}$.

$5n^2+3n+7 = 5(2k)^2 + 3(2k)+7$

$= 5(4k^2) + 3(2k)+7$

$= 10k^2 + 6k+7$

$= 2(5k^2+3k)+7$

$= 2m+7$ where $m = 5k^2+3k$

7 is an odd number while $2m$ is even meaning $5n^2+3n+7$ is odd.

Suppose n is odd, so $n = 2k+1$ for $k \in \mathbb{Z}$.

$5n^2+3n+7 = 5(2k+1)^2 + 3(2k+1)+7$

$= 5(4k^2+4k+1)+6k+3+7$

$= 10k^2+20k+5+6k+3+7$

$= 10k^2+26k+15$

$= 2(5k^2+13k)+15$

$= 2m+15$ where $m = 5k^2+13k$

15 is odd and $2m$ is even and adding an odd number to an even number makes it odd so $5n^2+3n+7$ is odd. ∎