Homework # 1

Read S-POP Part A and B (until ex. B(i)-1 on Pg. 3)
T-BOP sections 1.1 (Pg. 3-7) and 4.1-4.4 (Pg. 113-125)

Exercises:

- S-POP, Pt. B(i): Ex. 1, 2, 3
- T-BOP, Sec 1.1 (Pg. 7-8) Ex. A(1, 3, 6, 9, 14)
  B(17, 18, 26, 27)
  C(29, 32, 34, 38)
- T-BOP, Chap. 4 (Pg. 126-127): 5, 7, 14

My signal to end a proof: (it is a cat)

S-PoP Exercises:

B(i)-1 a) Proove that the sum of two odd numbers is even:

If n is an odd number, and m is an odd number, then $n+m$ is an even number.
Assume $n, m \ni \mathbb{Z}$ and both m and n are odd
Assume $n + m = l$ for some $l \ni \mathbb{Z}$.
because m and n are odd, say $m = 2k+1$ for some $k \ni \mathbb{Z}$ and $n = 2x+1$ for some $x \ni \mathbb{Z}$.
we can say $(2k+1) + (2x+1) = l$.

$$2k + 2x + 2 = l.$$

$$2(k + x + 1) = l.$$

the sum of integers are integers, so let $k + x + 1 = q$ for some $q \ni \mathbb{Z}$.

therefore, $2q = l$

therefore, by definition of an even number, the sum of 2 odd numbers is an even number.

B(i)-1 b) Prove that the product of 2 odd numbers is odd.

let $n, m \ni \mathbb{Z}$ be odd so $n = 2k+1$ and $m = 2x+1$ for some $k, x \ni \mathbb{Z}$.
let $mn = p$ for some $p \ni \mathbb{Z}$.

$$(2k+1)(2x+1) = p$$

$$4kx + 2k + 2x + 1 = p$$

$$2(2kx + k + x) + 1 = p$$

let $2kx + k + x = q$ for some $q \ni \mathbb{Z}$.

$$2q + 1 = p$$

So, by definition of an odd number, the product of 2 odd numbers is odd.

B(i) - 2

a) Proove that if n is even, then n^2 is divisible by 4

Suppose the even number $n = 2k$ for some $k \ni \mathbb{Z}$

let $n^2 = p$ for some $p \ni \mathbb{Z}$.

$$(2k)^2 = P$$

$$4k^2 = P$$

let $k^2 = Z$ for $z \ni \mathbb{Z}$

$$4Z = P$$

So, because $n^2 = 4z$, and $4z = P$, then the square of any even number, n, is divisible by 4.

b) Proove that if n is odd, then $n^2 - 1$ is divisible by 4.

Assume that because n is odd, $n = 2k+1$ for some $n, k \ni \mathbb{Z}$.

let $n^2 - 1 = l$ for $l \ni \mathbb{Z}$.

$$(2k+1)^2 - 1 = l$$

$$4k^2 + 4k + 1 - 1 = l$$

$$4(k^2 + k) = l$$

let $k^2 + k = x$ for $x \ni \mathbb{Z}$.

$$4x = l$$

Because $n^2 - 1 = l$ and $4x = l$ and $4x$ is divisible by 4, then $n^2 - 1$ is divisible by 4 for some odd integer n.

B(i) - 3 $a, b, c \ni \mathbb{Z}$. $a|b \Rightarrow$ a divisible by b if exists some $q \ni \mathbb{Z}$ such that $b = aq$

a) Prove that, if $a|b$ and $a|c$, then $a|(b+c)$

assume $a, b, c \ni \mathbb{Z}$

if $a|b$, then $b = aq$ for some $q \ni \mathbb{Z}$.

and if $a|c$ then $c = ax$ for some $x \ni \mathbb{Z}$

So, if we say $a|(b+c)$

$$= a \,|\, aq + ax$$

$$= a \,/\, a(q+x)$$

We know any product of an integer is divisible by that integer.

$$= a \,|\, a\left(\frac{b}{a} + \frac{c}{a}\right)$$

$$= a \,|\, (b+c)$$

therefore if $a|c$ and $a|b$, then $a|(b+c)$

B(i)-3

b) Proove that, if $a|b$, then $a|nb$ for $n \ni \mathbb{Z}$

$a, b, n \ni \mathbb{Z}$

by definition, if $a|b$, then $b = aq$ for some $q \ni \mathbb{Z}$.

so, $a \mid anq$.

any integer is divisible by the product of that integer and another integer, so by definition, if $a|b$ then $a|nb$ for some $n \ni \mathbb{Z}$

Note to self: cardinality/size denoted by $|x|$ or $|A|$ → only if $x|A$ is a set

set Builder $X = \{\text{expression} : \text{Rule}\}$ ($: \approx |$)

- T-BOP, Sec 1.1 (Pg. 7-8) Ex. A(1, 3, 6, 9, 14)
  B(17, 18, 26, 27)
  C(29, 32, 34, 38)
- T-BOP, Chap. 4 (Pg. 126-127): 5, 7, 14

A.

1) $\{5x-1 : x \in \mathbb{Z}\} = \{\dots, -11, -6, -1, 4, 9, \dots\}$

3) $\{3x+2 : x \in \mathbb{Z}\} = \{\dots, -4, -1, 2, 5, 8, \dots\}$

6) $\{x \in \mathbb{R} : x^2 = 9\} = \{-3, 3\}$

9) $\{x \in \mathbb{R} : \sin \pi x = 0\} = x \ni \mathbb{Z}$ or $\{\dots, -1, 0, 1, 2, \dots\}$

14) $\{5x : x \in \mathbb{Z}, |2x| \leq 8\} = \{\pm 20, \pm 15, \pm 10, \pm 5, 0\}$

$|x| \leq 4$
or
$-4 \leq x \leq 4$

B.

17) $\{2, 4, 8, 16, 32, 64 \dots\} = \{2^n : n \in \mathbb{N}\}$

18) $\{0, 4, 16, 36, 64, 100 \dots\} = \{4(n^2) : n \in \mathbb{Z}, n \geq 0\}$

26) $\{\dots, \frac{1}{27}, \frac{1}{9}, \frac{1}{3}, 1, 3, 9, 27, \dots\} = \{3^n : n \in \mathbb{Z}\}$

27) $\{\dots, -\pi, -\frac{\pi}{2}, 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi, \frac{5\pi}{2}, \dots\} = \{\frac{n\pi}{2} : n \in \mathbb{Z}\}$

C. (29, 32, 34, 38)

29) $|\{\{1\}, \{2, \{3, 4\}\} \varnothing\}| = 3$

32) $|\{\{\{1, 4\}, a, b, \{\{3, 4\}\}, \{\varnothing\}\}\}| = 1$

34) $|\{x \in \mathbb{N} : |x| < 10\}| = |\{1, 2, 3, 4, 5, 6, 7, 8, 9\}| = 9$

38) $|\{x \in \mathbb{N} : 5x \leq 20\}| = |\{x \in \mathbb{N} : x \leq 4\}| = |\{1, 2, 3, 4\}| = 4$

S - BoP) 5, 7, 14

5) Suppose $x, y \in \mathbb{Z}$. If x is even, then xy is even.

by definition of an even numbers, let $x = 2k$ for some $k \in \mathbb{Z}$.

let $xy = l$ for some $l \in \mathbb{Z}$

so, $y(2k) = l$

$2ky = l$

ky is the product of 2 integers, which is an integer.

so, by definition of an even number, xy is even

7) Suppose $a, b \in \mathbb{Z}$. If $a|b$, then $a^2|b^2$

We know that if $a|b$, $b = ak$ for some $k \ni \mathbb{Z}$.

So $a|ak$ and $a^2|a^2k^2$

$a^2 = a^2$ and $b^2 = a^2k^2$.

any number is divisible by its multiple.

So by defintion of divisibility,

$$a^2|a^2k^2 \Rightarrow a^2|b^2$$

14) If $n \in \mathbb{Z}$, then $5n^2 + 3n + 7$ is odd

Case 1: n is odd.

If n is odd, then $n = 2k+1$ for some $k \in \mathbb{Z}$.

So, $5(2k+1)^2 + 3(2k+1) + 7$

$= 5(4k^2 + 4k + 1) + 6k + 3 + 7$

$= 20k^2 + 20k + 5 + 6k + 10$

$= 20k^2 + 26k + 14 + 1$

$= 2(10k^2 + 13k + 7) + 1$

let $10k^2 + 13k + 7 = m$ where $m \in \mathbb{Z}$

$= 2m + 1$

by definition of an odd numbers, if $n \in \mathbb{Z}$, then $5n^2 + 3n + 7$ is odd

Case 2: n is even

If n is even, then let $n = 2k$ for $k \ni \mathbb{Z}$

$5(2k)^2 + 3(2k) + 7$

$= 5(4k^2) + 6k + 6 + 1$

$= 2(10k^2 + 3k + 3) + 1$

let $m = 10k^2 + 3k + 3$ for some $m \in \mathbb{Z}$

$= 2m + 1$

So, by definition of an odd number, if $n \in \mathbb{Z}$, then $5n^2 + 3n + 7$ is odd.