Hw 1

S-POP

B(i) - 1

Assume A and B are ~~to~~ two odd numbers
Then $A = 2m + 1$ for $m \in \mathbb{Z}$ and $B = 2n + 1$ for some $n \in \mathbb{Z}$

Hence $A + B = 2m + 1 + 2n + 1$

$A + B = 2(m + n + 1)$

~~Since~~ $A + B = 2 \cdot L$
where $L = m + n + 1$

So $A + B$ is even $\square$

B(i) - 2

a)

Assume n is even
Then by definition $n = 2k$ where $k \in \mathbb{Z}$
Hence $n^2 = (2k)^2$

$n^2 = 4k^2$

~~Since~~ so by definition $4 | n^2$ $\square$

b)

Assume n is odd
Then by definition $n = 2k + 1$ where $k \in \mathbb{Z}$
Hence

$n^2 - 1 = (2k + 1)^2 - 1$

$n^2 - 1 = 4k^2 + 4k + 1 - 1$

$n^2 - 1 = 4k^2 + 4k$

$n^2 - 1 = 4(k^2 + k)$

So by definition

$4 | n^2 - 1$ $\square$

B (1) b)

Let $x, y \in \mathbb{Z}$

Then by definition $x = 2k+1$ and $y = 2L+1$ where $k, L \in \mathbb{Z}$

Thus

$$x \cdot y = (2k+1)(2L+1)$$
$$= 4kL + 2k + 2L + 1$$
$$= 2(2kL + k + L) + 1$$
$$= 2m + 1$$

~~where~~

where $m = 2kL + k + L$ and $m \in \mathbb{Z}$

thus $x \cdot y$ is odd $\square$

B(i) - 3

a) Let $a, b, c \in \mathbb{Z}$ and
Assume $a | b$ and $a | c$
Then by definition $b = a \cdot g$ for $g \in \mathbb{Z}$ and
$c = a \cdot i$ for $i \in \mathbb{Z}$

~~Since~~ $\cancel{a | (b+c) = (a \cdot g) + (a \cdot i)}$

$\cancel{a | (b+c) = a(g+i)}$

Since $b + c = (a \cdot g) + (a \cdot i)$

$b + c = a(g+i)$ and that $(g+i) \in \mathbb{Z}$

we conclude that $a | (b+c)$ $\square$

b)

Let $a, b, c, n \in \mathbb{Z}$
Assume $a | b$
Then by definition $b = a \cdot m$ where $m \in \mathbb{Z}$

Since $bn = (a \cdot m) n$

$bn = a(m \cdot n)$ and $m \cdot n \in \mathbb{Z}$

we conclude that $a | nb$ $\square$

T-BOP

A

1 $\{5x - 1 : x \in \mathbb{Z}\}$

$\{\ldots, -11, -6, -1, 4, 9, \ldots\}$

~~3~~ $\cancel{\{x \in \mathbb{N} : -2 \le x < 7\}}$

$\cancel{\{\ldots, 1, 2, 3, 4, 5, 6\}}$

3) $\{x \in \mathbb{Z} : -2 \leq x \leq 7\}$

$\{-2, -1, 0, 1, 2, 3, 4, 5, 6\}$

6) $\{x \in \mathbb{R} : x^2 = 9\}$

$\{-3, +3\}$

9) $\{x \in \mathbb{R} : \sin \pi x = 0\}$

$\{\ldots, -2, -1, 0, 1, 2, \ldots\} = \mathbb{Z}$

14) $\{5x : x \in \mathbb{Z}, |2x| \leq 8\}$

$\{-20, -15, -10, -5, 0, 5, 10, 15, 20\}$

B

17) $\cancel{\{n \in \mathbb{N} : n^2}$

$\{x = 2^n : n \in \mathbb{N}, n \geq 1\}$

18)

$\{x = 4n^2 : n \in \mathbb{Z}, n \geq 0\}$

26) $\{x = 3^n : n \in \mathbb{Z}\}$

27) $\{x = \frac{\pi}{2} \cdot n : n \in \mathbb{Z}\}$

c)

~~|29.1| = 3 {1}, {2, {3,4}}~~

|32.| = 1 |29.1| = 3

|34.| = 9

|38.| = 4

~~|29.1| = {1}, {2, {3,4}}, {∅}~~

~~|32.1|~~

T-BOP

5)

Let $x, y \in \mathbb{Z}$

Assume x is even

Then by definition $x = 2k$ where $k \in \mathbb{Z}$

so

$x \cdot y = 2k \cdot y$

$xy = 2(ky)$ where $k \cdot y \in \mathbb{Z}$

so $x \cdot y$ is even $\square$

7)

let $x, y \in \mathbb{Z}$

Assume $a | b$

Then by definition $b = a \cdot m$ where $m \in \mathbb{Z}$

so $b^2 = (am)^2$

$b^2 = a^2 \cdot m^2$ where $m^2 \in \mathbb{Z}$

so $a^2 | b^2$ $\square$

14

let $n \in \mathbb{Z}$

case one n is even

then by definition $n = 2k$ where $k \in \mathbb{Z}$

so

$$5n^2 + 3n + 7 = 5(2k)^2 + 3(2k) + 7$$
$$= 20k^2 + 6k + 7$$
$$= 2 \cdot (10k^2 + 3k + 3) + 1$$
$$= 2 \cdot L + 1$$

where

$L = 10k^2 + 3k + 3$

so by definition $5n^2 + 3n + 7$ is odd

case two n is odd

then by definition $n = 2k + 1$ where $k \in \mathbb{Z}$

so

$$5n^2 + 3n + 7 = 5(2k+1)^2 + 3(2k+1) + 7$$
$$= 5(4k^2 + 4k + 1) + 6k + 4 + 7$$
$$= 20k^2 + 20k + 5 + 6k + 3 + 7$$
$$= 20k^2 + 26k + 15$$
$$= 2(10k^2 + 13k + 7) + 1$$
$$= 2 \cdot L + 1$$

where

$L = 10k^2 + 13k + 7$

then by definition $5n^2 + 3n + 7$ is odd $\square$