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Homework #1

S-POP B(i)-1 (a) Thm: The sum of two odd numbers is even.

Proof.

Consider two numbers a and b and assume a and b are odd. By the definition of an odd number, $a = 2n+1$ and $b = 2k+1$ such that $n, k \in \mathbb{Z}$. Thus, $a+b = 2n+2k+2 = 2(n+k+1)$. The term $n+k+1 \in \mathbb{Z}$ can be represented by m, so $a+b = 2m$. By the definition of an even number, the sum of arbitrary odd numbers a and b is even and hence we can say that the sum of two odd numbers is even.

(b) Thm: The product of two odd numbers is odd.

Proof.

Let a and b be arbitrary variables and assume both are odd. Thus, by definition of odd, $a = 2n+1$ and $b = 2k+1$ such that $n, k \in \mathbb{Z}$. $a \cdot b = (2n+1)(2k+1) = 4nk + 2n + 2k + 1 = 2(2nk + n + k) + 1$. The value $2nk + n + k$ is an integer as $n, k \in \mathbb{Z}$ and can then be represented by m, so $a \cdot b = 2m+1$. By the definition of odd, ab is an odd number, and because a and b were arbitrary, it follows that the product of two odd numbers is odd

B(i)-2 (a) Thm: If n is an even number, then n^2 is divisible by 4.

Proof.

Consider an arbitrary even number n. By definition, $n = 2k$ for some $k \in \mathbb{Z}$. Thus, $n^2 = (2k)^2 = 4k^2$. It can then be said that n^2 is divisible by 4, as $4 | 4k^2$. Hence, if n is even, then $4 | n^2$.

(b) Thm: If n is an odd number, then $n^2 - 1$ is divisible by 4.

Proof.

Let n be an arbitrary odd number. By definition of odd $n = 2k-1$ for $k \in \mathbb{Z}$. It can then be said that $n^2 - 1 = (2k-1)^2 - 1 = 4k^2 - 4k + 1 - 1 = 4(k^2 - k)$. Because k is an integer, the term $k^2 - k = m$ for $m \in \mathbb{Z}$. Hence, $n^2 - 1 = 4m$, and $4 | 4m$ because 4m is a multiple of 4, so it follows that $4 | n^2 - 1$. Thus, if n is odd, $n^2 - 1$ is divisible by 4.

B(i)-3 (a) Thm: If $a|b$ and $a|c$, then $a|(b+c)$.
Assume $a|b$ and $a|c$ for some $a, b, c \in \mathbb{Z}$. By definition of divides, there exists $q, p \in \mathbb{Z}$ such that $b = aq$ and $c = ap$. It follows that $b + c = aq + ap = a(q+p)$. We can say $q + p = m$ for some $m \in \mathbb{Z}$, so $b + c = a.m$. By definition of divides, we can then say $a | b+c$. Thus, if $a|b$ and $a|c$, it is true that $a|(b+c)$.
(b) Thm: If $a|b$, then $a|nb$ for any integer n.
Let a, b be integers such that $a|b$. By definition of divides, $b = aq$ for some $q \in \mathbb{Z}$. Consider an arbitrary integer n. $n.b = a \cdot q \cdot n$. The value $q.n$ can be represented by m for some $m \in \mathbb{Z}$. Thus, we can say that $nb = am$, and it follows from the definition of divides that $a|nb$. So, if $a|b$, then for any integer n, $a|nb$.

T-BOP A) 1. $\{\ldots, -1, 4, 9, 14, \ldots\}$ 3. $\{-2, -1, 0, 1, 2, 3, 4, 5, 6\}$

6. $\{3\}$ 9. $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$

14. $\{-20, -15, -10, -5, 0, 5, 10, 15, 20\}$

B) 17. $\{2^x : x \in \mathbb{N}\}$ 18. $\{4x^2 : x \geq 0\}$

26. $\{3^x : x \in \mathbb{Z}\}$ 27. $\{\frac{\pi}{2}x : x \in \mathbb{Z}\}$

C) 29. 3 32. 1 34. 9 38. 4

$\boxed{\text{Ch. 4}}$ 5. Thm: If x is even, then xy is even.
Proof.
Let $x, y \in \mathbb{Z}$ and assume x is even. By definition of even, $x = 2n$ for some $n \in \mathbb{Z}$, so $xy = 2n \cdot y$. The term ny can be represented by arbitrary integer m, and thus $xy = 2m$. So by the definition of an even number xy is even, and we can conclude that if x is even, it follows that xy is even.

7. Thm: For $a, b \in \mathbb{Z}$, if $a|b$, then $a^2|b^2$.

Proof.

Let a and b be arbitrary integers such that $a|b$. By definition of divides, it follows that $b = aq$ for some $q \in \mathbb{Z}$. Thus, $b^2 = (aq)^2 = a^2q^2$. Let $q^2 = m$ for some $m \in \mathbb{Z}$. We can then say $b^2 = a^2 \cdot m$, from which we can say $a^2|b^2$ by definition of divides. Thus, for $a, b \in \mathbb{Z}$, if $a|b$, then $a^2|b^2$.

14. Thm: If $n \in \mathbb{Z}$, then $5n^2+3n+7$ is odd.

Proof.

Assume $n \in \mathbb{Z}$ and consider the following 2 cases:

a) Assume n is odd. By definition of odd, $n = 2k+1$ for some $k \in \mathbb{Z}$. It follows that $5n^2+3n+7 = 5(2k-1)^2+3(2k-1)+7 = 20k^2-20k+5+6k-3+7 = 20k^2-14k+9$. We can say $20k^2-14k = m$ for some $m \in \mathbb{Z}$. Thus, $5n^2+3n+7 = m+9$, and 9 added to any integer m would make it odd. So, if n is odd, $5n^2+3n+7$ is odd.

b) Assume n is even. $n = 2k$ for some $k \in \mathbb{Z}$ by definition of even. Thus, $5n^2+3n+7 = 5(2k)^2+3(2k)+7 = 20k^2+6k+7$ where $20k^2+6k$ is an integer that can be represented by m. So, $5n^2+3n+7 = m+7$, and odd integer 7 added to another integer produces an odd value. Hence if n is even, $5n^2+3n+7$ is odd.

Because arbitrary integer n can only be even or odd and $5n^2+3n+7$ produces an odd value in both cases, if $n \in \mathbb{Z}$ it follows that $5n^2+3n+7$ is odd.