Homework 1

Wednesday, January 17, 2024 3:35 PM

S-POP B(i) 1, 2, 3

T-BOP 1.1 A(1, 3, 6, 9, 14)

B(17, 18, 26, 27)

C(29, 32, 34, 38)

T-BOP Chp 4 (126–127) (5, 7, 14)

- B(i) - 1 - A
- Assume the sum of two odd numbers is even. And $n \in \mathbb{R}$.
- By definitions of even and odd numbers:
- Then $(2n+1)+(2n+1) = 2K \quad K \in \mathbb{R}$

$= 4n+2 = 2K$

$= 2(2n+1) = 2K$

Let $l = 2n+1$

$2l = 2K$

Therefore, by definition of an even number. The sum of two odd numbers is even. □ Tschüss. ← my end signature!

- B(i) - 1 - B
- Assume the product of two odd numbers is odd. And that $n \in \mathbb{R}$.
- By the definition of odd numbers:

$(2n+1)\cdot(2n+1) = 2n+1$

then, $4n^2+4n+1 = 2n+1$

$2(2n^2+2n)+1 = 2n+1$

Let $K \in \mathbb{R}$

$= 2K+1 = 2n+1$

Therefore, The product of two odd numbers is odd. □ Tschüss.

- B(i) - 2 - A
- Assume n is even and n^2 is divisible by 4. And $x \in \mathbb{R}$. We also know if n^2 is even, then n is even
- By the definition of even numbers, we can set this up as

$(2x)^2 = n$

$4x^2 = n$

Therefore, we can see that n^2 divides 4. □ Tschüss.

- B(i) - 2 - B
- Assume that n is an odd number and $n^2-1 \mid 4$, also $n \in \mathbb{R}$. We also know if n^2-1 is odd, $n-1$ is odd
- By the definition of an odd number

$n = 2K+1$ where $K \in \mathbb{R}$

so $n^2 = 4K^2+4K+1$

$n^2-1 = 4(K^2+K)$

So we can see that n^2-1 divides 4.

□

B(i) - 3 - A

Assume if $a \mid b$ and $a \mid c$ then $a \mid (b+c)$

- By definition, there must be an integer K $(K \in \mathbb{Z})$ such that $b = aK$ so that $a \mid b$
- And, there must be an integer l $(l \in \mathbb{Z})$ such that $c = al$ so $a \mid b$
- So, $b+c = al + aK$ which equals $a(l+K)$
- Therefore a divides $(b+c)$ □

B(i) - 3 - B

- Assume that if $a \mid b$ then $a \mid nb$ for $n \in \mathbb{Z}$
- By definition, if $a \mid b$ then there is some n such that $b = an$
- So we see that $b = an$, so it follows that

$a \mid b \Rightarrow a \mid an$ □

- A - 1

$\{\ldots -6, -1, 4, 9, 14 \ldots 5x-1\}$

- A - 3

$\{-2, -1, 0, 1, 2, 3, 4, 5, 6\}$

- A - 6

$\{-3, 3\}$

- A - 9

$\{\ldots -2, -1, 0, 1, 2 \ldots\}$

- A - 14

$\{-20, -15, -10, -5, 0, 5, 10, 15, 20\}$

- B - 17

$\{2x : x \in \mathbb{Z},\ x > 0\}$

- B - 18

$\{x^2 : x \in \mathbb{Z}\}$

- B - 26

$\{3^x : x \in \mathbb{Z}\}$

- B - 27

$\{\pi x : x \in \mathbb{Z}\}$

- C - 29

3

- C - 32

1

- C - 34

$\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

9

- C - 38

$\{1, 2, 3, 4\}$

4

- Chapter 4 - 5
- Assume if x is even, then xy is even and $x, y \in \mathbb{Z}$
- By definition, $x = 2n$ where $n \in \mathbb{R}$
- So, $(2n)y = 2ny$
- By definition $2ny$ must be even
- Therefore, xy is even □ Tschüss.

- Chapter 4 - 7
- Assume $a \mid b$ and $a, b, n \in \mathbb{Z}$
- By the definition of divisibility, there has to be some n so that $b = an$
- So, $b^2 = (an)^2 \qquad n^2 = K \quad K \in \mathbb{Z}$

$b^2 = a^2 n^2$

- Therefore $b^2 = a^2 d$ which by definition means $a^2 \mid b^2$

□ Tschüss.

- Chapter 4 - 14
- Assume $n \in \mathbb{Z}$
- Consider 2 cases
  - Case 1: $5n^2+3n+7$ is odd

$n = 2K+1$ by def of odd numbers $K \in \mathbb{Z}$

$5(2K+1)^2 + 3(2K+1) + 7$

$5(4K^2+4K+1) + 6K+3+7$

$20K^2 + 26K + 15$

$2(10K^2+13K)+15 \qquad 10K^2+13K = l \quad l \in \mathbb{Z}$

$2l+15$ is odd by definition of an odd number

  - Case 2: $5n^2+3n+7$ is even

$n = 2K$ by def of even numbers $K \in \mathbb{Z}$

$5(2K)^2 + 3(2K) + 7$

$20K^2 + 6K + 7$

$2(10K^2+3K)+7 \qquad l = 10K^2+3K \quad l \in \mathbb{Z}$

$2l+7$

$2l+7$ is NOT even by definition.

- Therefore $5n^2+3n+7$ is odd.

□ Tschüss.