Solutions to Selected Exercises, HW #4

Assignment from T-BOP:

- Section 1.6, pages 20–21: Exercises 1(acdfi), 2(befi), 4.
- Section 1.7, pages 22–24 : Exercises 3, 4, 6, 8, 11, 14.
- Section 1.8, pages 24–28: Exercises 1–6, 9, 10, 11.

T-BOP, Section 1.6

Exercise 2. Let $A = \{0, 2, 4, 6, 8\}$ and $B = \{1, 3, 5, 7\}$ have universal set $U = \{0, 1, 2, ..., 8\}$. Find:

(b) $\overline{B} = \{0, 1, 2, ..., 8\} - \{1, 3, 5, 7\} = \{0, 2, 4, 6, 8\} = A$.

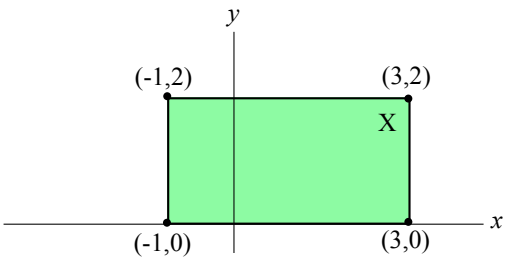
(e) $A - \overline{A} = \{0, 2, 4, 6, 8\} - \{1, 3, 5, 7\} = \{0, 2, 4, 6, 8\} = A$.

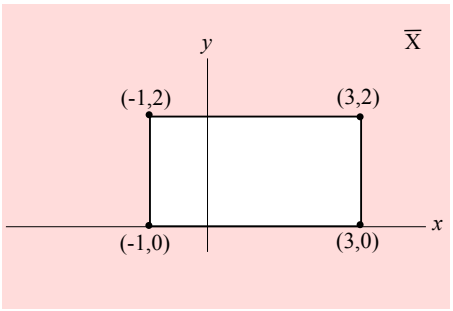
(f) $\overline{A \cup B} = \{0, 1, 2, ..., 8\} - (\{0, 2, 4, 6, 8\} \cup \{1, 3, 5, 7\}) = \emptyset$.

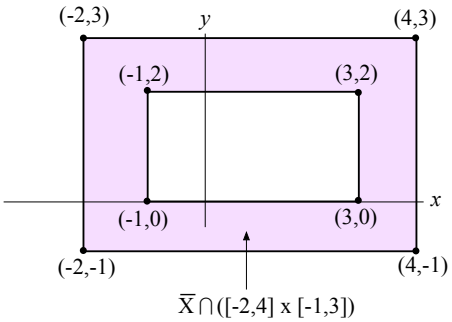
(i) $\overline{\overline{A} \cap B} = \{0, 1, 2, ..., 8\} - (\{1, 3, 5, 7\} \cap \{1, 3, 5, 7\}) = \{0, 2, 4, 6, 8\} = A$.

Exercise 4. Sketch the set $X = [-1, 3] \times [0, 2]$ on the plane $\mathbb{R}^2$. On separate drawings, shade in the sets $\overline{X}$ and $\overline{X} \cap ([-2, 4] \times [-1, 3])$.

SOLUTION:



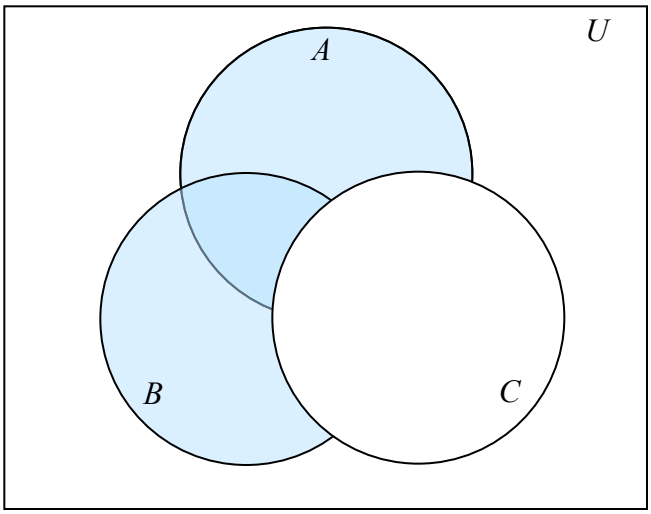




T-BOP, Section 1.7

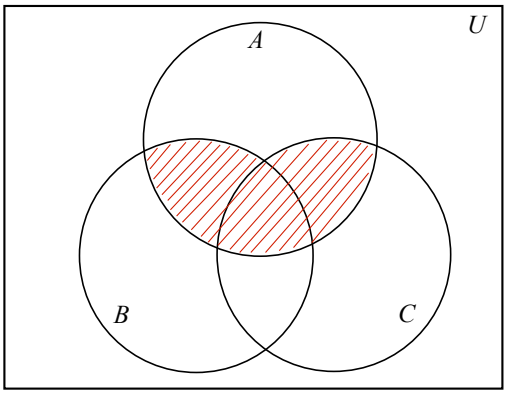
Exercise 4. Draw a Venn diagram for $(A \cup B) - C$.

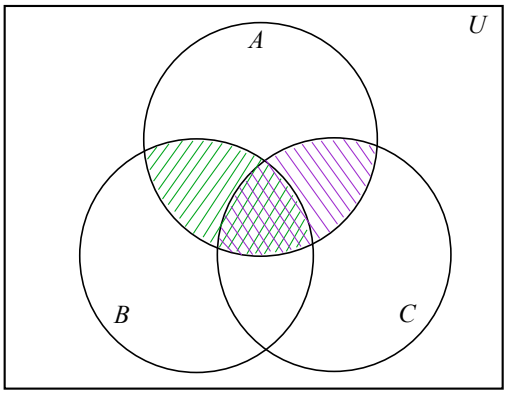
SOLUTION:



Exercise 6. Draw Venn diagrams for $A \cap (B \cup C)$ and $(A \cap B) \cup (A \cap C)$. Based on your drawings, do you think $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$?

SOLUTION:





Yes, it sure seems that way. In the Venn diagram on the left, $A \cap (B \cup C)$ is shaded in red. In the Venn diagram on the right, $(A \cap B)$ is shaded in green, and $(A \cap C)$ is shaded in purple. Certainly the region shaded in the left Venn diagram looks like the total region shaded in the right one. (In fact, we proved in class that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ for all sets A, B, C.)

Exercise 14. The expression for the set shown is $(A \cap B \cap C) \cup (A - (B \cup C))$.

T-BOP, Section 1.8

Exercise 2.

$$\bigcup_{i=1}^{3} A_i = \{0, 2, 3, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24\} \quad \text{and} \quad \bigcap_{i=1}^{3} A_i = \{0, 12, 24\}.$$

Exercise 4.

$$\bigcup_{i\in\mathbb{N}} A_i = 2\mathbb{Z} \quad \text{and} \quad \bigcap_{i\in\mathbb{N}} A_i = \{0\}.$$

Exercise 6.

$$\bigcup_{i\in\mathbb{N}} [0, i+1] = [0, \infty) \quad \text{and} \quad \bigcap_{i\in\mathbb{N}} [0, i+1] = [0, 2].$$

Exercise 10.

$$\bigcup_{x\in[0,1]} [x, 1]\times[0, x^2] = \{(x, y) \in [0, 1]\times[0, 1] : y \leq x^2\} \quad \text{and} \quad \bigcap_{x\in[0,1]} [x, 1]\times[0, x^2] = \{(1, 0)\}.$$